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Automation and Remote Control

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SCIENTIFIC AND ENGINEERING PROBLEMS OF AUTOMATED ELECTRIC DRIVES DURING THE CURRENT SEVEN-YEAR PERIOD

M. G. Chilikin and I. I. Petrov

The far-reaching plan for the development of the national economy of the USSR in the period 1959-1965, put forth at the Twenty-first Congress of the Communist Party of the Soviet Union, envisions an all-around increase in the productivity of national labor on the basis of a strengthening of its electrical sinews and the automation of production. The electrical motive power available determines the degree to which production is mechanized, since the basic and principal form of the prime mover for operating machinery is the electric drive. The productivity of labor depends directly on its electrification (i.e., on the electric motive power available to it): over the past ten years, the productivity of industry in the USSR increased by about the same percentage as its electrification.

By an electric drive we understand a complex device which transforms electrical energy to mechanical energy, and which provides electrical control of the transformed mechanical energy. In this connection, the future development of electric drives must be directed to the implementation of all forms of motion without the use of any mechanical means in achieving these purposes (rotary and reciprocating motion; prolonged discontinuous variable and tracking motion; with fixed speed, and with speeds which change sharply, which vary continuously in accordance with a given law, or which change arbitrarily). The development of electric drives must provide for the broadening of their functions in the automatic control and regulation of the operating machines of technological aggregates (control of the starting, reversing, and braking of operating machines; maintaining constancy of speed, torque and power; program control of operating machines, etc.).

The science of electric drives derives basically from the work of Soviet scientists and, today, this branch of technical knowledge has taken definitive shape in the form of an independent section of applied electrical engineering. In the Soviet Union there has been created, and introduced into industry, a system of electric drives of a high technological level. However, the total volume of research work carried out by research and planning institutes, by higher educational institutions, by the design offices of enterprises and other organizations is still insufficient. There remain unsolved many problems concerning electric drives which are of great significance for the country's national economy.

Among the unsolved problems, the first that should be mentioned is the creation of controlled economical ac electric drives. Of particular significance here are the questions of developing simple and reliable systems of frequency control for ac electric motors and, in particular, of creating ac supplies with controlled frequency. Here, special attention must be given to the development and investigation of static-frequency transformers using ionic, electronic, and semiconductor technologies.

The situation is somewhat better with the development of controlled ac electric drives with choke control. In the last few years there have been created, and introduced into industry, a number of such electric drives, and the scientific and engineering methods for designing them have been adequately developed. However, the domain of possible applicability of electric drives with choke control has been somewhat overestimated, which sometimes leads to incorrect choices in selecting the type of electric drive for productive machines. Thus, for example, on insufficient grounds, a choked electric drive was used for excavators of medium productivity, which led to their uneconomical and unreliable performance. Special resolutions of the State Scientific-Engineering Committee of the Soviet Ministers of the USSR recommended that the production of excavators with such electric drives be halted, and that they be replaced by controlled dc electric drives. The attempt to use choked electric drives for materials-handling vehicles was also adjudged unsuccessful. Seemingly, it is necessary to find reasonable limits of applicability of choke-controlled electric drives, taking into account their relatively low energy efficiency for tight and prolonged control of production-machine speed, and the peculiarities of these machine's load characteristics.

Also requiring further investigation are the questions of the dynamics of choked electric drives, with the nonlinearities of their individual elements taken into account, and also the question of electric-drive heating as the load varies cyclically.

In connection with the intensification of productive processes, the necessity arises for developing electric drives with asynchronous short-circuited electric motors designed for production mechanisms with partial starts and reverses. However, this question remains unsolved. In the near future there must be created, based on normally manufactured asynchronous short-circuited electric motors, a modification of the electric motor with a heat-insulating base and with an extended motor, designed for a high frequency of switching in, and there should also be developed a methodology for designing electric drives with partial starts and reverses. With this, particular at-

tention should be given to the transient responses in electric drives, and to the question of their heating.

There has been an improvement recently in the status of the investigations into the operating properties of ac electric drives supplied from autonomous supplies of commensurable power. Even here, however, there remain many problems in connection with increasing the power of asynchronous electric motors supplied from individual generators. The questions as to the dynamics of the processes transpiring in such electric drives deserve particular attention.

It is also going to be necessary to investigate the questions related to the use of synchronous electric motors for uncontrolled (operating at constant speed) and controlled (operating with variable speed) industrial electric drives. The transition to circuits with direct starting of synchronous motors and starting with permanently connected exciters led to a certain broadening of the domain of applicability of synchronous motors. However, despite their specific properties and their high energy efficiency, synchronous motors have still not been as widely used as they should be. Still, a wide dissemination of synchronous electric motors for drives of operational machines can significantly increase the power factor of electrical stands and provide a high rigidity of electric-drive mechanical characteristics within a wide range of speeds. The use of motors with rotary stators permits the problem of synphasing a synchronous shaft to be solved, etc. There might, therefore, be great interest in works devoted to elucidating new domains of applicability of synchronous electric motors, and to the investigation of new automatic control systems for synchronous electric drives, operating both with constant and, especially with variable speeds of rotation. In particular, interest inheres in the questions of the economic speed control of synchronous electric motors with frequency control, and in questions of automatic excitation control of electric motors for drives with impact loads. Also necessary are further investigation of the transient responses in synchronous electric drives and the development of a scientific method of calculating these processes.

Many problems in the domain of controlled dc electric drives also require solution. In the widely used "motor-generator" (M-G) systems, only dc motors with parallel excitation are used, these being less liable to torque overheating than motors with series and with hybrid excitation. The use of power semiconducting rectifiers, connected in the series-excitation winding circuit, permits one to create new types of M-G systems with improved dynamic characteristics. Analogous systems can also be created by supplying the motor from controlled mercury-arc rectifiers (CMR).

Significant success has been achieved in the development and dissemination of electric drives with ionic transformers (for controlled nonreversible drives). An essential improvement of the dynamic characteristics of an M-G drive system has been attained by the use of controlled mercury-arc rectifiers for the excitation of the system's machine. However, the development and dissemination of such systems have been carried out extremely slowly.

Great difficulties ensue when a CMR-M system is used for reversible electric drives. In this connection, great interest inheres in the question of further development and investigation of an automatic control system for an ionic electric drive, where the ionic transformer must be considered as an element of the over-all control system. These systems must possess a high energy factor, permit operation in the rectifying and inverting modes, be fast-acting, and have little control power.

Development and investigation are required for ionic-drive systems, increasing the power factor for tight speed control, and also for systems for the automatic control of the power factor of ionic aggregates for the purpose of providing the very best conditions for supplying electricity to enterprises.

Recently, magnetic amplifiers have been finding increasing application in electric drives. They are used in "magnetic amplifier – motor" electric drive systems, in M-G systems for generator excitation control, for line control by mercury-arc rectifiers, and also as industrial amplifying links.

The employment of semiconductor amplifiers has also been broadened, these being used advantageously in the form of preamplifier stages in conjuction with rotary and magnetic amplifiers. However, the domain of application of these new types of amplifiers has still not been defined sufficiently clearly. It is necessary to carry out a resolute replacement of rotary amplifiers in electric-drive systems by the more reliable and long-lasting magnetic and semiconductor amplifiers.

Simultaneously, it is necessary to work on the creation of power semiconductor rectifiers for "semiconductor amplifier – motor" electric-drive systems, and also for the control of generator excitation in M-G systems. The new means of automation (rotary and magnetic amplifiers, semiconductor and electronic devices) allow the creation of highly efficient electric-drive control systems, providing the maximum usage of the machines, the accurate maintaining of a given trajectory of motion, etc. Of particular significance here are the so-called optimal systems in which, given limitations on trajectory, speed, acceleration, or heating, there is attained a minimum time of execution of the drive mechanism's operating function.

The investigation of the transient responses in such essentially nonlinear systems presents significant difficulty. Today we are completely capable of solving these complicated tasks by the method of mathematic modeling using analog and digital computers. However, for the creation of automated electric-drive systems, these capabilities are used very weakly. This is explained, on the one hand, by the insufficient quantities of such machines at the disposal of research and planning organizations and,

on the other hand, by the shortage of scientific and engineering cadres familiar with the methods of mathematical modeling. The preparation of such cadres is unsatisfactory in the higher institutions of learning which turn out engineers with the "electric drive" specialty. Until now, all the textbooks on electric drives have been based on the classical methods of investigation and do not expound, in adequate measure, the more modern methods, including the methods of mathematical and physical modeling. The laboratory courses in the educational institutions do not provide work with computers. In connection with this, the important problems requiring solution are: equipping research and educational institutes, planning-designing, and other organizations with analog and digital computer-models; improving the knowhow of the scientific and engineering cadres in the domains of mathematical and physical modeling; further developing of research work on the study of transient responses in complex systems of automated electric drives, constructed with the use of the new means of automation, and on the synthesis of the modern, essentially nonlinear, systems which provide optimal electric-drive transient responses; modernization of the university courses in "electric drives" and the related disciplines in the direction of developing in them the more modern methods of theoretical and experimental investigation of electric drives; the creating, in the near future, of manuals and textbooks on electric drives which are designed to make the transition from the older methods of studying electric-drive dynamics to the newer ones, including the methods of mathematical and physical modeling.

Certain problems in the domain of multimotor electric drives are also not solved. Thus, further investigation is required of electric shaft systems with variable speed ratios of the individual elements, for mass-production-line electric drives. It is necessary to develop a theory of the nonsteady-state processes in electric drives with friction coupling between the motors, and to investigate questions of oscillation damping in electric shaft systems with frequency transformers and dual-source machines. There has yet to be created a generalized theory for multimotor electric drives with electrical connections, providing a constant tension on the material being processed, for aggregates with continuous technological processes.

Insufficient attention has been given to the production, and to the use in automated electric drives, of the various electromagnetic clutches, including those with ferromagnetic packing. Nonetheless, their use could lead to the creation of a rational electric-drive system, particularly in cases where it is necessary to have smooth starting of the drive, acceleration of large inertial masses, control of torque, speed, etc. They can also play the roles, in automated electric drive systems, of brake dynamometer, position indexer, torque limiter, etc. Today, electromagnetic clutches are comparatively widely used only in machine-stand construction. It is necessary to translate the experience in machine-stand construction to other branches of the national economy, and to carry out a number of investigative tasks, directed toward the explication of the possibilities of constructing rational electric-drive systems with electromagnetic clutches for diverse productive mechanisms.

During many years, various branches of industry, and particularly the machine-building industry, have put forward the problem of creating electric drives with rotary-reciprocating and vibratory motion. Unfortunately, this problem is still unsolved for practical purposes. Research organizations must necessarily increase the exploratory work in the domains of creating reciprocating electric motors, and of constructing autooscillatory systems with simple control systems for the amplitude and frequency of the vibrations.

Of great interest in both the theory and the practice of electric drives might be an investigation of systems containing elastic links. Today, all system elements are considered to be ideally solid bodies, and therefore the phenomena and processes in which elastic deformations play a part are not taken into consideration. The deformations of elastic links lead, in many cases, to significant errors. It would thus be of great advantage to develop general methods of analyzing automatic-control systems containing elastic links, and to solve the problem of synthesizing electric drives and their control systems for mechanisms with elastic links.

Despite the significant development of this theory in recent years, certain questions in the theory of electric-drive heating are still unsolved. Thus, further investigation is needed of the heating processes of ac motors in normal condtions of use, including the asymmetric mode of supply of three-phase asynchronous electric motors. Also requiring further development are the questions of electric-motor heating in transient responses and for cyclical loads, with account being taken of insulation aging. Work should also be continued on the direct use of heat-conduction equations for the thermal computations for electric-drive motors.

There are also many questions in the domain of microdrives to be resolved. This domain is little studied but, at the same time, this energetically evolving. In the next few years, millions of electric motors will be required for microdrives. In connection with this, there is necessary a further development of the questions of creating economical systems of actual microdrives, of increasing their reliability and noiselessness in use, and also the accuracy and speed of response of microdrives as used in the devices of automation.

The modern electric drive plays a very large role in systems of automated productive processes. The particularly essential question is that of program control of the electric drives of individual productive mechanisms and aggregates. Today, a certain measure of success has been achieved in the introduction of program-control systems for a number of metal-working stands, in the screw-down devices of reversible rolling mills, in the electric drives

of charging systems for blast furnaces, etc. However, the general state of work in the areas of creating and disseminating program control systems is unsatisfactory. In these systems, the new means of automation – semiconducting and magnetic elements – are used with inadequate completeness, and one finds little reflection of the recent achievements in the area of creating self-adjusting and self-learning systems. As a rule, the programmed systems in use today have a rigidly given program, which does not always answer to the required technological-economical indicators of the process under conditions of nonconstant external disturbances. The use of devices providing automatic program correction in correspondence with changes in an aggregate's external operating conditions would permit the creation of an automatic-control system which would establish the most efficient mode of operation

A significant obstacle to the widespread introduction of automatic control by mechanisms supplied with electric drives is the inadequacy of existing position transducers of the machines' individual organs and of the material to be processed, and also the inadequacy of manufactured relay-contact apparatus which allows only a limited number of switchings in. Thus, the number of contact operations on an automatic machine-stand line reaches tens of thousands, and sometimes even hundreds of thousands, per hour, which sharply reduces the reliability of electric control systems. In this connection, great importance inheres in the development of new types of contactless automatic control devices which use highly reliable semiconducting and magnetic elements.

Insufficient use is made, in control schemes with drives in production lines, of electric apparatus used for communciations devices. It is necessary to practice a wider use of this apparatus which has small dimensions and increased reliability.

For the purpose of accelerating the large-scale use of modern automatic-control systems by electric drives, it becomes necessary to accelerate the utilization, in the electrical industry's standard assemblies, of new control systems, including digital and programmed devices, and also complex control systems for standard electric drives.

The enumeration of problems in the domain of electric drives which has been given here is far from exhaustive, and contains only the first-priority problems.

STATISTICAL THEORY OF GRADIENT SYSTEMS OF AUTOMATIC OPTIMIZATION FOR OBJECTS WITH QUADRATIC CHARACTERISTICS

A. A. Fel'dbaum
(Moscow)

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Original article submitted June 12, 1959

The paper formulates the problem of investigating gradient systems of automatic optimization in the presence of random factors. Methods are given for determining the steady-state error and estimating the duration of the transient response for gradient systems with several input variables. The results are illustrated by examples of systems with one and with two variables.

INTRODUCTION

The near future will be marked by a significant development of self-adjusting (adaptive) systems. Great value, therefore, inheres in the development of a theory of automatic serach, which is the basic process in such systems, and which differs essentially from the process of error liquidation in automatic-control systems. One of the classes of adaptive systems which promises to be of importance practically is that of automatic optimization systems. We present below the theory of one of the forms of such systems.

The problem of an automatic optimization system consists of automatically finding and maintaining a minimum of some quantity $\underline{y}$ with, generally speaking, additional conditions imposed. The quantity $\underline{y}$ is a function of the variables u_i:

$$y = y(u_1, u_2, \ldots, u_m). \qquad (1)$$

The process of automatic search for a minimum is slowed when there is random noise present, and the minimum itself will be determined, and maintained, with some error under these circumstances. The duration of the transient response of the search process, and the steady-state error, are the fundamental symptomatic indicators of the system's process.

The most general problem consists of synthesizing such a controlling portion which will implement the best (in some sense) process of automatic search. We shall consider below the more restricted problem of determining the indicators of the process for a given controlling portion, and also of choosing the optimal values of this portion's parameters.

In the automatic serach for a minimum, tentative movements are made – "experiments" with the object, as it were – and thereafter the operational movement is made on the basis of an analysis of the results of the tentative trial motions. Various combinations of operationa and trial movements are possible. In the sequel, we consider systems of the discrete type in which, during one cycle, a series of trial steps is taken, and then followed by an operational step; in the following cycle, there is again taken a series of trial steps followed by an operational step, etc.

Figure 1 shows an example of an automatic optimization system. Here, C is the controlling portion; at the output of part F_0 of object 0 there is the quantity $\underline{y}$ which must be minimized. It is joined, in part F of the object, by an error $\underline{z}$. At the object's input, the controlling quantities u_i are joined by random-noise quantities, z_{0i}. The sums thus obtained

$$x_i = u_i + z_{0i} \qquad (2)$$

are applied to the input of part F_0 of the object. In carrying out trials on the $\underline{i}$th input variable, one adds to the quantity x_i the quantity $+\alpha_{0i}$ in the first trial interval and $-\alpha_{0i}$ in the second trial interval. Thus, a trial consists of two intervals. If the total number of input variables is $\underline{m}$, the entire cycle is composed of 2m intervals. We shall assume that the variations of the object's input quantities require the expenditure of no time, but that the carrying out of an "experiment," as a result of which the quantity $\underline{y}$ appears at the object's output, requires some time interval of constant duration.

The presence of the noise z_{0i} at the input gives rise to a random displacement of the minimum point $\underline{y}$ in the space $u_1, u_2, \ldots, u_m$, which imitates a slow "drift" of the characteristics, which occurs in actual objects. It is possible to set $z_{0i} = \gamma_i n$, where $\underline{n}$ is discrete time or the number of cycles ($n = 0, 1, 2, \ldots$) and the γ_i are random variables. The noise $\underline{z}$ at the object's output (for example, in a measuring block) is a random noise of comparatively high frequency. We shall suppose that the increments of these quantities are independent random variables with known distribution law $\psi(v)$, zero mean value and finite dispersion σ^2. Such a form for the noise was introduced for these systems in [1].

The Markov Process in a Gradient System of Automatic Optimization

Let the dependence of the output y' of portion F_0 of the object on the input quantities x'_k be described by the formula ·

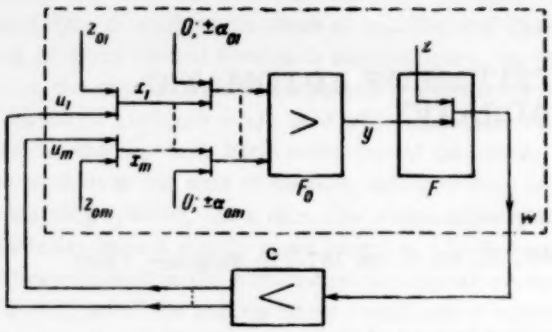

Fig. 1.

$$y' = \sum_{i,\,k=1}^{m} b'_{ik}\, x'_i\, x'_k + \sum_{k=1}^{m} c'_{ik}\, x'_k + d'. \qquad (3)$$

By applying a linear transformation to each variable separately, we can take this formula to the form

$$y = f(x_1, \ldots, x_m) = \sum_{i,\,k=1}^{m} b_{ik}\, x_i\, x_k, \qquad (4)$$

where $b_{ik} = b_{ki}$. We shall assume that quadratic form (4) is positive definite. Then y has a unique minimum at the origin of coordinates.

During the ith pair of intervals, whose ordinal numbers within the cycle are j and $j+1$, where $j = 2i-1$, measurements are made of the quantity y. We denote the values of the quantities x_i in the jth interval by $x_i^{(j)}$, and the value of y by $y^{(j)}$. In the first interval of the ith pair, there is the quantity $x_i^{(j)} + \alpha_{0i}$ at the ith input, while the second interval finds the quantity $x_i^{(j+1)} - \alpha_{0i}$ at this input.

After these two intervals, we determine the difference

$$\Delta_i y_n = f(x_{1,n}^{(j)}, \ldots, x_{i,n}^{(j)} + \alpha_{0i}, \ldots, x_{m,n}^{(j)}) - f(x_{1,n}^{(j+1)}, \ldots, x_{i,n}^{(j+1)} - \alpha_{0i}, \ldots, x_{m,n}^{(j+1)}). \qquad (5)$$

The subscripts n denote that the measurements were made in the nth cycle. The corresponding quantity at the output of controlling portion C (Fig. 1) is defined by the expression

$$\Delta_i w_n = \Delta_i y_n + \Delta_i z_n = \Delta_i y_n + v_i. \qquad (6)$$

The gradient method consists of this: that after a series of trials, as a result of which one determines the direction of the gradient of the quantity y – the vector with components $\partial y/\partial x_i \cong \Delta_i y_i$, the operational step is taken in this direction. Obviously, the increments of the quantities $u_{i,n+1} - u_{i,n}$, where n is discrete time, must be proportional to the quantities $\Delta_i w$, found in the nth cycle:

$$u_{i,\,n+1} - u_{i,\,n} = -\alpha_i(\Delta_i w_n) = -\alpha_i(\Delta_i y_n + v_i). \qquad (7)$$

By taking into account (2) and (5), and the formula $z_{0i} = \gamma_i n$, we find that

$$x_{i,\,n+1} = x_{i,\,n} + \gamma_i - \alpha_i [f(x_{1,n}^{(2i-1)}, \ldots, x_{i,n}^{(2i-1)} + \alpha_{0i}, \ldots, x_{m,n}^{(2i-1)}) - f(x_1^{(2i)}, \ldots, x_i^{(2i)} - \alpha_{0i}, \ldots, x_m^{(2i)}) + v_i] \qquad (8)$$

Here, we have denoted $x_{1,n}^{(1)}$ simply by $x_{1,n}$. Since

$$x_i^{(j)} = x_i^{(1)} + \gamma_i \frac{j-1}{2m} = x_i + \gamma_i \frac{j-1}{m},$$

$$x_i^{(j+1)} = x_i + \gamma_i \frac{2i-1}{2m}, \qquad (9)$$

then

$$x_{i,\,n+1} = x_{i,\,n} + \gamma_i - \alpha_i \Big[f\Big(x_1 + \gamma_1 \frac{i-1}{m}, x_2 + \gamma_2 \frac{i-1}{m}, \ldots, x_i + \gamma_i \frac{i-1}{m} + \alpha_{0i}, \ldots, x_m + \gamma_m \frac{i-1}{m}\Big) - f\Big(x_1 + \gamma_1 \frac{2i-1}{2m}, \ldots, x_i + \gamma_i \frac{2i-1}{2m} - \alpha_{0i}, \ldots, x_m + \gamma_m \frac{2i-1}{2m}\Big) + v_i\Big]. \qquad (10)$$

For brevity, we denote $x_{1,n} = \xi_1$ and $x_{1,n+1} = x_1$; we introduce the vector $\underline{x}$ in the m-dimensional space Ω_x, the vector ξ with coordinates ξ_i in the space $\Omega_\xi = \Omega_x$ and the vector γ with coordinates γ_i and the vectors ξ_{0i}, again in space Ω_x. Then (10) can be rewritten in the form

$$x_i = \xi_i + \gamma_i - \alpha_i \{ f(\xi + \alpha_{0i}\xi_{0i} + \gamma \frac{i-1}{m}) - f(\xi - \alpha_{0i}\xi_{0i} + \gamma \frac{2i-1}{2m}) + v_i \}. \qquad (11)$$

We now take the form of function f(x) [cf., (4)] into account. By substituting (4) in (10) or (11), we obtain after some transformation,

$$x_i = \gamma_i - \alpha_i v_i + \sum_{j=1}^{m} A_{ij}\, \xi_j + B_i, \qquad (12)$$

where

$$A_{ij} = \delta_{ij} + \frac{\alpha_i}{m} \sum_{\substack{k=1 \\ k \neq i}}^{m} b_{kj}\, \gamma_k + \alpha_i b_{ji} \Big(\frac{\gamma_i}{m} - 4\alpha_{0i} \Big),$$

$$B_i = \frac{4i-3}{2m} \alpha_i \Big\{ \frac{1}{2m} \sum_{\substack{j,\,k=1 \\ j,\,k \neq i}}^{m} b_{jk}\, \gamma_j \gamma_k + \Big(\frac{\gamma_i}{m} - 2\alpha_{0i} \Big) \sum_{\substack{j=1 \\ j \neq i}}^{m} b_{ji}\, \gamma_j + b_{ii}\, \gamma_i \Big(\frac{\gamma_i}{2m} - 2\alpha_{0i} \Big) \Big\}. \qquad (13)$$

Here, $\delta_{ij} = 1$ for $i = j$ and $\delta_{ij} = 0$ for $i \neq j$; i.e., δ_{ij} is a Kronecker delta.

The sequence $x_{1,n}$ ($n = 0, 1, 2, \ldots$), or the vector x_n, is a random Markov process (cf. [2,3]) since the probability distribution for x_{n+1} depends on the value of x_n.

Initially, let the γ_i be fixed quantities. In view of the independence of the γ_i and the v_i, we can compute the distribution $W(\gamma)$ for γ at any subsequent stage. Further, let the v_i now be all zero. We first investigate the convergence of the automatic search process when the random noises are lacking. Equation (12) can be rewritten as:

$$\mathbf{x} = A\boldsymbol{\xi} + \mathbf{B}, \qquad (14)$$

where A is the matrix with elements A_{ij}, and B is the vector with coordinates $\gamma_i + B_i$. If the system converges to a steady state x_{ste}, then this latter state may be found from the equation

$$x_{ste} = Ax_{ste} + B. \tag{15}$$

Let E be the unit matrix, and let the inverse $(E-A)^{-1}$ of the matrix E-A exist. Then

$$x_{ste} = (E - A)^{-1} B. \tag{16}$$

We consider the differences

$$x_{n+1} - x_{ste} = \Delta_{n+1}, \quad x_n - x_{ste} = \Delta_n. \tag{17}$$

If follows, from (14), (16), and (17) that

$$\Delta_{n+1} = A\Delta_n. \tag{18}$$

The necessary and sufficient conditions for the convergence of the search process, i.e., for the holding of the condition $\Delta_n \to 0$ as $n \to \infty$, are the inequalities

$$|\lambda_k| < 1, \tag{19}$$

where the λ_k $(k = 1, \ldots, m)$ are the roots of the characteristic equation $|A - E\lambda| = 0$ of the matrix A. As is well known, these inequalities reduce to condtions of the Routh-Hurwitz type. We shall assume that these conditions are satisfied.* With this, the speeds γ_i can not be arbitrary; the vector γ must lie within a certain region Ω_γ of the m-dimensional space.

We turn now to the general equation, in which random noise is present. Let the transition probability from the point $\xi = \{x_{1,n}, x_{2,n}, \ldots, x_{m,n}\}$ in the interval $x_i \le x_{i,n+1} \le x_i + dx_i$ $(i = 1, \ldots, m)$ equal $\varphi(\xi, x)$. $dx_1 \ldots dx_m = \varphi(\xi, x)\, d\Omega_x$, where $d\Omega_x = dx_1, \ldots, dx_m$.

Then, $p_{n+1}(x)$, the probability density that x_{n+1} appear in the interval $x_i \le x_{i,n+1} \le x_i + dx_i$ $(i = 1, \ldots, m)$, is defined by the formula

$$p_{n+1}(x) = \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} \varphi(\xi, x)\, p_n(\xi)\, d\xi_1 \ldots d\xi_m = \int_{\Omega_\xi} \varphi(\xi, x)\, p_n(\xi)\, d\Omega_\xi, \tag{20}$$

where $d\Omega_\xi = d\xi_1, \ldots d\xi_m$. If this linear integral transformation has a fixed point to which the sequence $p_n(x)$ tends, then by denoting

$$P(x) = \lim_{n\to\infty} p_n(x), \tag{21}$$

we may find this function from the linear integral equation

$$P(x) = \int_{\Omega_\xi} (\xi, x) P(\xi)\, d\Omega_\xi. \tag{22}$$

We find $\varphi(\xi, x)$, assuming now that the γ_i are fixed. From (12) we get

$$v_i = \frac{1}{\alpha_i}\left[\gamma_i - x_i + \sum_{j=1}^{m} A_{ji}\,\xi_j + B_i\right]. \tag{23}$$

The conditional probability that $x_{i,n+1}$ is found in the interval $x_i \le x_{i,n+1} \le x_i + dx_i$ if $x_{j,n} = \xi_j$ $(j = 1, \ldots, m)$ equals $\psi(v_i)\, dx_i / \alpha_i$, where the v_i are determined from (23). Since, for different x_i, the v_i are independent, we have that

$$\varphi(\xi, x) = \prod_{i=1}^{m} \frac{\psi(v_i)}{\alpha_i} = \prod_{i=1}^{m} \frac{1}{\alpha_i}\psi\left\{\frac{1}{\alpha_i}\left[\gamma_i - x_i + \sum_{j=1}^{m} A_{ij}\,\xi_j + B_i\right]\right\}. \tag{24}$$

Determination of the Indicators of the Automatic Search Process

We now find the mathematical expectation of the random variable y_n, by which we understand the arithmetic mean over the nth cycle:

$$M[y_n] = \int_{\Omega_x} \frac{1}{2m}\sum_{i=1}^{m}\left[f\left(x + \alpha_{0i}\xi_{0i} + \gamma\frac{i-1}{m}\right) + f\left(x - \alpha_{0i}\xi_{0i} + \gamma\frac{2i-1}{2m}\right)\right] p_n(x)\, d\Omega_x. \tag{25}$$

We consider separately the expression in the square brackets. By substituting (2) for f(x) we obtain, after some transformations,

$$\frac{1}{2m}\sum_{i=1}^{m}\left[f\left(x + \alpha_{0i}\xi_{0i} + \gamma\frac{i-1}{m}\right) + f\left(x - \alpha_{0i}\xi_{0i} + \gamma\frac{2i-1}{2m}\right)\right] = \sum_{j,k=1}^{m} b_{jk}\, x_j x_k + \left(1 - \frac{1}{2m}\right)\sum_{j,k=1}^{m} b_{jk}\,\lambda x_{jk} + C, \tag{26}$$

where

$$C = \frac{\frac{2}{3}(m+1)(4m-7)+5}{8m^2}\sum_{j,k=1}^{m} b_{jk}\gamma_j\gamma_k + \frac{1}{m}\sum_{i=1}^{m} b_{ii}\,\alpha_{0i}^2 - \frac{1}{2m^2}\sum_{i,j=1}^{m} b_{ij}\,\alpha_{0i}\gamma_j. \tag{27}$$

Therefore,

$$M[y_n] = \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\left\{\sum_{j,k=1}^{m} b_{jk}\, x_{j,n}\, x_{k,n} + \left(1 - \frac{1}{2m}\right)\sum_{j,k=1}^{m} b_{jk}\, x_{j,n}\,\gamma_k + C\right\} + p_n(x_{1,n}; x_{2,n}; \ldots; x_{m,n})\, dx_{1,n}\, dx_{2,n} \ldots dx_{m,n} \tag{28}$$

or

$$M[y_n] = \sum_{j,k=1}^{m} b_{j,k}\int_{\Omega_x} x_j x_k\, p_n(x)\, d\Omega_x + \left(1 - \frac{1}{2m}\right)\sum_{j,k=1}^{m} b_{jk}\,\gamma_k\int_{\Omega_x} x_j\, p_n(x)\, d\Omega_x + C, \tag{29}$$

* The stability conditions for continuous gradient systems, derived in [6], have well-known similarities to those given above for $\gamma_i = 0$.

since

$$\int_{\Omega_x} p_n(\mathbf{x})\, d\Omega_x = 1.$$

We introduce the notation

$$I_{jk,\,n} = \int_{\Omega_x} x_j x_k\, p_n(\mathbf{x})\, d\Omega_x,$$

$$J_{j,\,n} = \int_{\Omega_x} x_j\, p_n(\mathbf{x})\, d\Omega_x. \tag{30}$$

Then,

$$M[y_n] = \sum_{j,\,k=1}^{m} b_{jk} I_{jk,\,n} + \left(1 - \frac{1}{2m}\right) \sum_{j,\,k=1}^{m} b_{jk}\gamma_k J_{j,\,n} + C. \tag{31}$$

Equations (30) for the integrals can be found from (20). We multiply both members by $x_j x_k d\Omega_x$ and integrate over Ω_x. Then, $I_{jk,\,n+1}$ appears in the left member of the equation, and the expression takes the form:

$$I_{jk,\,n+1} = \int_{\Omega_x} x_j x_k \left[\int_{\Omega_\xi} \varphi(\xi, \mathbf{x})\, p_n(\xi)\, d\Omega_\xi\right] d\Omega_x = \int_{\Omega_x} x_j x_k \left[\int_{\Omega_\xi} \prod_{i=1}^{m} \frac{\psi(v_i)}{\alpha_i}\, p_n(\xi)\, d\Omega_\xi\right] d\Omega_x. \tag{32}$$

We now reverse the order of integration, and change from the variables $x_1, x_2, \ldots, x_m$ to the new integrand variables $v_1, v_2, \ldots, v_m$. Since $dx_i = -\alpha_i dv_i$, and the change in sign of the differential compensates for the changes in the signs of the limits of integration, we then find, by letting $d\Omega_v = dv_1 \ldots dv_m$, that

$$I_{jk,\,n+1} = \int_{\Omega_\xi} p_n(\xi) \left[\int_{\Omega_v} \sum_{i=1}^{m} \psi(v_i) (\gamma_j - \alpha_j v_j + \sum_{\nu=1}^{m} A_{j\nu}\xi_\nu + B_j)(\gamma_k - \alpha_k v_k + \sum_{\nu=1}^{m} A_{k\nu}\xi_\nu + B_k)\, d\Omega_v\right] d\Omega_\xi = \int_{\Omega_\xi} p_n(\xi) L_{jk}\, d\Omega_\xi. \tag{33}$$

We now consider the interior integral L_{jk} included within the square brackets. By expanding the parentheses in it, we obtain

$$L_{jk} = \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} \left[\prod_{i=1}^{m} \psi(v_i)\right] [\alpha_k \alpha_j v_k v_j + v_j(-\alpha_j\gamma_k - \alpha_j \sum_{\nu=1}^{m} A_{k\nu}\xi_\nu - \alpha_j B_k) + v_k(-\alpha_k\gamma_j - \alpha_k \sum_{\nu=1}^{m} A_{j\nu}\xi_\nu - \alpha_k B_j) + (\gamma_j\gamma_k + \gamma_k B_j + \gamma_j B_k + B_k B_j) + (\gamma_j + \gamma_k) \sum_{\nu=1}^{m} (A_{k\nu} + A_{j\nu})\xi_\nu + (B_j + B_k) \sum_{\nu=1}^{m} (A_{k\nu} + A_{j\nu})\xi_\nu + \sum_{\mu,\,\nu=1}^{m} A_{k\nu} A_{j\mu}\xi_\mu\xi_\nu] dv_1, \ldots, dv_m. \tag{34}$$

Now, we take into account that

$$\int_{-\infty}^{+\infty} \psi(v_i)\, dv_i = 1, \qquad \int_{-\infty}^{+\infty} v_i \psi(v_i)\, dv_i = 0, \tag{35}$$

and denote the dispersion of the noise at the output by

$$\sigma^2 = \int_{-\infty}^{+\infty} v_i^2 \psi(v_i)\, dv_i. \tag{36}$$

By taking (35) and (36) into account, we may rewrite (34) in the form

$$L_{jk} = \delta_{jk}\alpha_k^2\sigma^2 + (\gamma_j\gamma_k + \gamma_j B_k + \gamma_k B_j + B_k B_j) + \sum_{\nu=1}^{m} [(\gamma_j + B_j) A_{k\nu} + (\gamma_k + B_k) A_{j\nu}]\xi_\nu + \sum_{\mu,\,\nu=1}^{m} A_{k\nu} A_{j\mu}\xi_\mu\xi_\nu. \tag{37}$$

By substituting (37) in (33), we obtain

$$I_{jk,\,n+1} = \delta_{jk}\alpha_k^2\sigma^2 + (\gamma_j\gamma_k + \gamma_j B_k + \gamma_k B_j + B_j B_k) + \sum_{\nu=1}^{m} [(\gamma_j + B_j) A_{k\nu} + (\gamma_k + B_k) A_{j\nu}] J_{\nu,\,n} + \sum_{\mu,\,\nu=1}^{m} A_{k\nu} A_{j\mu} I_{\mu,\nu} \tag{38}$$

It is possible to write $m(m+1)/2$ such equations for determining $I_{jk,\,n+1}$ from $I_{\mu\nu,\,n}$. The m quantities $J_{\nu,\,n}$ ($\nu = 1, \ldots, m$) also appear in these equations. These latter quantities can be found from (20) in an analogous way. Indeed, by multiplying this expression by $x_{j,\,n+1} d\Omega_x$ and integrating over Ω_x, we arrive at the formula

$$J_{j,\,n+1} = \int_{\Omega_x} x_j \left[\int_{\Omega_\xi} \varphi(\xi, \mathbf{x})\, p_n(\xi)\, d\Omega_\xi\right] d\Omega_x = \int_{\Omega_x} x_j \left[\int_{\Omega_\xi} \prod_{i=1}^{m} \frac{\psi(v_i)}{\alpha_i}\, p_n(\xi)\, d\Omega_\xi\right] d\Omega_x = \int_{\Omega_\xi} p_n(\xi) \left[\int_{\Omega_x} x_j \prod_{i=1}^{m} \frac{\psi(v_i)}{\alpha_i} d\Omega_x\right] d\Omega_\xi = \int_{\Omega_\xi} p_n(\xi) L_j\, d\Omega_\xi \tag{39}$$

With this,

$$L_j = \int_{\Omega_x} x_j \prod_{i=1}^{m} \frac{\psi(v_i)}{\alpha_i} d\Omega_x = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \prod_{i=1}^{m} \psi(v_i)\,(\gamma_j - \alpha_j v_j + \sum_{\nu=1}^{m} A_{j\nu}\xi_\nu + B_j)\, dv_1 \ldots dv_m = \gamma_j + \sum_{\nu=1}^{m} A_{j\nu}\xi_\nu + B_j. \tag{40}$$

By substituting (40) in (39), we find that

$$J_{j,\,n+1} = \int_{\Omega\xi} p_n(\xi)\,(\gamma_j + \sum_{\nu=1}^{m} A_{j\nu}\xi_\nu + B_j)\,d\Omega_\xi = (\gamma_j + B_j) + \sum_{\nu=1}^{m} A_{j\nu} J_{\nu,\,n}. \qquad (41)$$

Using (41), we can determine $J_{j,\,n+1}$ from the $J_{\nu,\,n}$, and then substitute the result in (38). The presence of the noises v_i is completely unreflected in (41). For $v_i = 0$, the distribution densities reduce to unit pulse functions, but (41) remain unchanged by this. Therefore, the necessary and sufficient conditions for convergence for the sequence $J_{\nu,\,n}$ as $n \to \infty$ remain the same inequalities (19). The sequence of second moments $I_{jk,\,n}$ of the distribution also converges for $v_i \neq 0$.

In fact, the presence of the noise v_i in (38) is reflected by the additive terms $\delta_{jk}\,\alpha_k^2\,\sigma^2$. For $\sigma^2 = 0$ (absence of noises v_i), conditions (19) are necessary and sufficient for convergence of the sequence $I_{jk,n}$. It is obvious that the presence of the additional terms $\delta_{jk}\,\alpha_k^2\sigma^2$ only changes the steady-state value, but has no effect on the fact of convergence, since the transformation matrix of the $I_{jk,\,n}$ [cf. (38)] depends only on the A_{jk}. Therefore, inequalities (19), when random noises v_i are present, also guarantee convergence of the first and second moments and, consequently, convergence of the sequence $M[y_n]$ [cf. (31)].

We set

$$I_{jk} = \lim_{n\to\infty} I_{jk,\,n}; \quad J_j = \lim_{n\to\infty} J_{j,\,n};$$

$$\eta_i = \lim_{n\to\infty} M[y_n]. \qquad (42)$$

It follows, from (31), (38), and (41), that the quantities I_{jk}, J_j, and η are defined by (43), (44), and (45):

$$I_{jk} = \delta_{jk}\alpha_k^2\sigma^2 + (\gamma_j\gamma_k + \gamma_j B_k + \gamma_k B_j + B_j B_k) + \sum_{\nu=1}^{m} [(\gamma_j + B_j)A_{k\nu} + (\gamma_k + B_k)A_{j\nu}]J_\nu + \sum_{\mu,\,\nu=1}^{m} A_{k\nu}A_{j\mu}I_{\mu,\,\nu} \quad (j,\,k = 1,\ldots,m). \qquad (43)$$

$$J_j = (\gamma_j + B_j) + \sum_{\nu=1}^{m} A_{j\nu}J_\nu \quad (j = 1,\ldots,m), \qquad (44)$$

$$\eta_i = \sum_{j,\,k=1}^{m} b_{jk}I_{jk} + \left(1 - \frac{1}{2m}\right)\sum_{j,\,k=1}^{m} b_{jk}\gamma_k J_j + C. \qquad (45)$$

Thus, the problem of determining the mathematical expectation η of the quantity y in the steady-state mode is solved. A consequence of (43), (44), and (45) is that the error is defined only by the dispersion σ^2 of the noise, and does not depend on the form of the function $\psi(v)$. This property is a consequence of the quadratic characteristic of the object. The mathematical expectation of the deviations from the minimum (in the given case, $y_{min} = 0$) in the steady state may be considered as a certain measure of the error in this mode. As for the duration of the transient response, its mathematical expectation may also serve as its measure. This criterion was introduced in [4].

However, another measure turns out to be simpler — not the mathematical expectation of the duration of the transient response, but the duration of the transient response for the mathematical expectation, which is not the same thing. Consider the dependence of $M[y_n]$ on n [cf. (31)]. This is a transient response in some equivalent discrete system which may be studied by the ordinary methods, for example by frequency methods, integral methods, etc. In the simple cases (cf., below), one can obtain an expression for the duration of the transient response in such equivalent systems.

If the vector γ is not fixed, but has some probability distribution $W(\gamma)$, we may then, by denoting $d\Omega_\gamma = d\gamma_1 \ldots d\gamma_m$, average the values of η over Ω_γ in accordance with the formula

$$\eta_{iav} = \int_{\Omega_\gamma} \eta_i W(\gamma)\,d\Omega_\gamma. \qquad (46)$$

It may be more advantageous for computational purposes to use (45), where the maximum expected values of the γ_i are substituted.

Example of a Simplest System (m = 1)

We illustrate the use of the formulas obtained by the example of a simple system with one input variable. We set

$$\alpha_1 = \alpha,\ \alpha_{01} = \alpha_0,\ x_1 = x,\ \xi_1 = \xi,\ \gamma_1 = \gamma,\ b_{11} = b,\ B_1 = B,\ A_{11} = A.$$

In this case, (19) for search stability takes the form $|A| < 1$ or, in accordance with (13),

$$|A| = |1 - 4\alpha\alpha_0 b + \alpha\gamma b| < 1. \qquad (47)$$

It is clear from this that the speed γ must not go beyond certain limiting values, i.e., $\gamma_1 \leq \gamma \leq \gamma_2$, where $(\gamma_1, \gamma_2) > 0$.

Equation (45) takes the form

$$\eta_i = bI_{11} + \frac{b\gamma}{2}J_1 + b\left(\alpha_0^2 - \gamma\frac{\alpha_0}{2} + \frac{\gamma^2}{8}\right). \qquad (48)$$

The integrals I_{11} and J_1 are determined from (43) and (44), which are rewritten in the form

$$I_{11} = \alpha^2\sigma^2 + (\gamma + B)^2 + 2A(\gamma + B) + A^2 I_{11}, \qquad (49)$$

$$J_1 = \gamma + B + AJ_1. \qquad (50)$$

Here, in accordance with the second of Eqs. (13),

$$B = -\alpha b\gamma\frac{1}{2}\left(2\alpha - \frac{\gamma}{2}\right) = \alpha b\gamma\left(\frac{\gamma}{4} - \alpha_0\right). \qquad (51)$$

From these expressions, we find that

$$J_1 = \frac{\gamma + B}{1 - A}, \quad I_{11} = \frac{\alpha^2\sigma^2 + (\gamma + B)^2}{1 - A^2} + 2A\frac{(\gamma + B)^2}{(1 - A)(1 - A^2)} \qquad (52)$$

The transient response is determined by (31), (38), and (41), which take the following form:

$$\eta_n = bI_{11,n} + b\left(\alpha_0^2 - \frac{\gamma\alpha_0}{2} + \frac{\gamma^2}{8}\right) + \frac{b\gamma}{2} J_{1,n},$$
$$I_{11,n+1} = \alpha^2\sigma^2 + (\gamma + B)^2 + 2A(\gamma + B) J_{1,n} + A^2 I_{11,n},$$
$$J_{1,n+1} = \gamma + B + AJ_{1,n}. \tag{53}$$

We set

$$A_0 = \alpha_0\sqrt{\frac{b}{\sigma}}, \quad \delta = \gamma\sqrt{\frac{b}{\sigma}}, \quad k = \alpha\alpha_0 b, \quad \varepsilon = \frac{\eta}{\sigma}. \tag{54}$$

Then, after substitution of the values of I_{11} and J_1 from (49) and (50), (48) assumes the form

$$\varepsilon = \left(A_0 - \frac{\delta}{4}\right)^2 + \left(\frac{\delta}{k}\right)^2 \frac{1}{\left(4 - \frac{\delta}{A_0}\right)^2} + \left(\frac{k}{A_0}\right)^2 \frac{1}{1 - \left(1 - 4k + \frac{k\delta}{A_0}\right)^2}. \tag{55}$$

We rewrite stability condition (47) in the form

$$\left|1 - 4k + k\frac{\delta}{A_0}\right| < 1. \tag{56}$$

Если $j = 1 - 4k + k\frac{\delta}{A_0}$, то $k\left(4 - \frac{\delta}{A_0}\right) = 1 - j = q$.

If $j = 1 - 4k + k\frac{\delta}{A_0}$, then $k\left(4 - \frac{\delta}{A_0}\right) = 1 - j = q$.

In a stable system, $-1 < j < 1$ and $0 < q < 2$. The boundary of stability is reached for

$$K_{bo} = \frac{2}{4 - \delta/A_0} \tag{57}$$

The other boundary is obtained from the condition that $k > 0$, i.e.,

$$4 - \frac{\delta}{A_0} > 0. \tag{58}$$

Thus, the region of stability has the form shown in Fig. 2.

As an example, let us set $\delta = 0.2$. For this value, and by giving A_0, we find from (55) the minimal value ε_{min} as a function of k. Let it be attained for $k = k_{opt}$ (Fig. 3). If we trace the curve $\varepsilon_{min} = \varepsilon_{min}(A_0)$ (Fig. 4a) and the curve $k_{opt} = k_{opt}(A_0)$ (Fig. 4b), we can find the minimum possible value of the steady-state error $\varepsilon_{\min\min}$ and the values of the parameters A_0 and k which correspond to it. It is clear from Fig. 4 that $\varepsilon_{\min\min} \approx 0.3$, $A_0 = 0.4$, and $k = 0.15$.

The optimal parameter values depend on the fact that for very large steps the error increases due to the significant deviations from the minimum; however, for very small steps the system can not compensate the effect of the "drift" γ of the characteristics, thanks to which the error again increases. One easily convinces oneself that

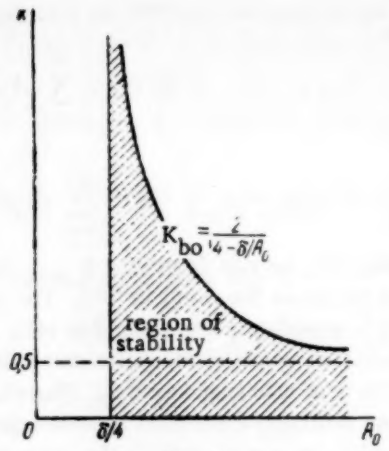

Fig. 2.

for $\gamma = 0$, the optimal values of α and α_0 are infinitesimally small and, in the limit, the error ε tends to zero.

We now find the duration of the transient response where, for simplicity, we limit ourselves to the case when $\gamma = 0$. Then from (51) and (53), we find that $B = 0$ and

$$\eta_n = bI_{11,n} + b\alpha_0^2, I_{11,n+1} = \alpha^2\sigma^2 + A^2 I_{11,n}, A = 1 - 4k. \tag{59}$$

The steady-state values turn out to be

$$I_{11} = \frac{\alpha^2\sigma^2}{8k(1-2k)}, \quad \eta = \frac{b\alpha^2\sigma^2}{8k(1-2k)} + b\alpha_0^2. \tag{60}$$

Since

$$I_{11,n} = \alpha^2\sigma^2 + A^2 I_{11,n-1} = \alpha^2\sigma^2 + A^2[\alpha^2\sigma^2 + A^2 I_{11,n-2}] = \cdots$$
$$\cdots = \alpha^2\sigma^2[1 + A^2 + \cdots + A^{2(n-1)}] + A^{2n} I_{11,0} =$$
$$= \alpha^2\sigma^2 \frac{1 - A^{2n}}{1 - A^2} + A^{2n} I_{11,0} = I_{11}(1 - A^{2n}) + A^{2n} I_{11,0}, \tag{61}$$

the difference

$$\theta_n = \eta_n - \eta = b(I_{11,n} - I_{11}) = bA^{2n}(I_{11,0} - I_{11}). \tag{62}$$

It may be that at the initial moment $I_{11,0} < I_{11}$, if the initial value of x is, for example, at the origin of coordinates. However, if we assume that the system is significantly distant from the steady-state position at the initial moment of time, we can assume that $I_{11,0} - I_{11} > 0$. Then, the quantity θ_n tends to zero with increasing n, while remaining positive. We now find the discrete time n_0 after whose expiration the following relationship is valid:

$$\frac{\theta_n}{\lambda} \leqslant 1, \tag{63}$$

where the constant $\lambda \ll 1$ is some admissable threashold.

It follows from (60), (62), and (63), that

$$n_0 = \frac{1}{2} \frac{\ln\lambda(I_{11} + \alpha_0^2) - \ln(I_{11,0} - I_{11})}{\ln A} \tag{64}$$

The quantity n_0 serves as a measure of the duration of the transient response in the system.

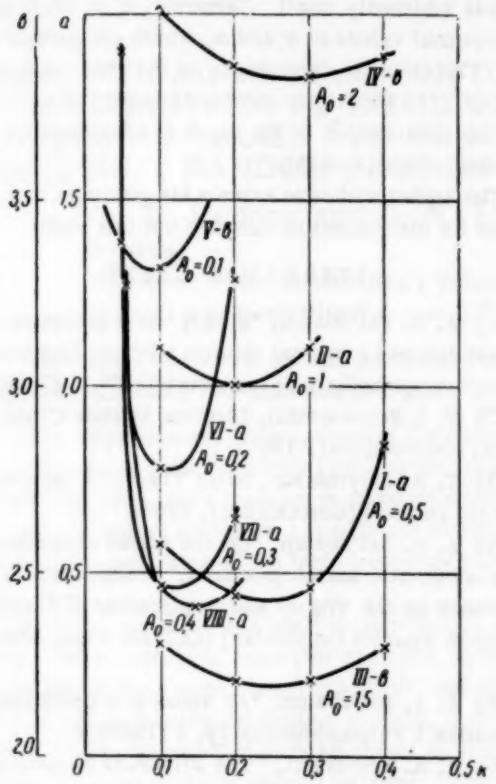

Fig. 3.

Example of a System with Two Variables (m = 2)

Let the object's equation have the form:

$$y = x_1^2 + 2\beta x_1 x_2 + x_2^2. \tag{65}$$

For $|\beta| < 1$, the unique minimum of the function y is found at the origin of coordinates. This case corresponds to m = 2, $b_{11} = b_{22} = 1$, and $b_{12} = \beta$ in the general formulas. We further set $\gamma_i = 0$, $\alpha_1 = \alpha_2 = \alpha$, $\alpha_{01} = \alpha_{02} = \alpha_0$, $\alpha_1\alpha_{01} = \alpha_2\alpha_{02} = k$. Then, from (13), we find that

$$A_{11} = A_{22} = 1 - 4k, \quad A_{12} = A_{21} = -4\beta k. \tag{66}$$

Equation (45) takes the form:

$$\eta = I_{11} + 2\beta I_{12} + I_{22} + \alpha_0^2, \tag{67}$$

and I_{11}, I_{12}, and I_{22} are found from (43), which can be written in the following forms:

$$\begin{aligned} &I_{11}8k(1-2k) + 8\beta k(1-4k)\,I_{12} - 16\beta^2k^2I_{22} = \alpha^2\sigma^2,\\ &I_{11}4\beta k(1-4k) + 8k(1-2k-2\beta^2k)\,I_{12} + I_{22}4\beta k(1-4k) = 0\\ &-16\beta^2k^2I_{11} + 8\beta k(1-4k)\,I_{12} + I_{22}8k(1-2k) = \alpha^2\sigma^2. \end{aligned} \tag{68}$$

In these equations, the quantities I_{11} and I_{22} enter in identical fashion, as was to be expected. We set

$$8\beta k(1-4k) = D, \quad 8k(1-2k-2\beta^2k^2) = E. \tag{69}$$

Then, solving (68), we find that

$$I_{11} = I_{22} = \frac{\alpha^2\sigma^2 E}{E^2 - D^2}, \quad I_{12} = \frac{\alpha^2\sigma^2 D}{D^2 - E^2} \tag{70}$$

and, as follows from (67),

$$\eta = \frac{2\alpha^2\sigma^2(E - \beta D)}{E^2 - D^2} + \alpha_0^2. \tag{71}$$

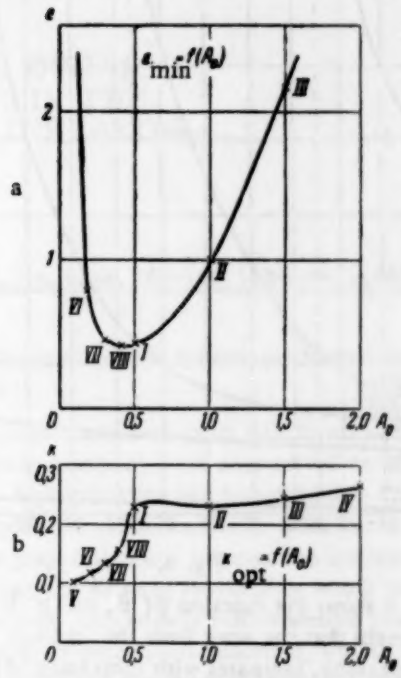

Fig. 4.

We now introduce the relative error from the random noise (for this, we assume that $\alpha_0\alpha = k$)

$$\frac{\eta - \alpha_0^2}{\left(\frac{\sigma}{\alpha_0}\right)^2} = \frac{2k^2(E - \beta D)}{E^2 - D^2} = \theta(k, \beta). \tag{72}$$

By substituting the values of E and D from (69), we find that

$$\theta(k, \beta) = \frac{k(1-2k)}{4} \times \frac{1-\beta^2}{(1-2k)^2 - 4\beta^2k(1-2k) + 4\beta^4k^2 - \beta^2(1-4k)^2}. \tag{73}$$

We find the stability conditions from the characteristic equation $|A - E\lambda| = 0$, or

$$\begin{vmatrix} A_{11} - \lambda & A_{12} \\ A_{21} & A_{22} - \lambda \end{vmatrix} = 0. \tag{74}$$

By substituting the values of A_{ij} here, we arrive at the equation

$$(1 - 4k - \lambda)^2 - 16\beta^2k^2 = 0, \tag{75}$$

whose roots are

$$\lambda_{1,2} = (1 - 4k) \pm 4\beta k. \tag{76}$$

The search is stable if $|\lambda_{1,2}| < 1$. It follows from this that k must be found within the limits

$$0 < k < \frac{1}{2(1 + |\beta|)}. \tag{77}$$

For a given k, the quantity β may vary within the limits

$$|\beta| < \frac{1}{2k} - 1. \tag{78}$$

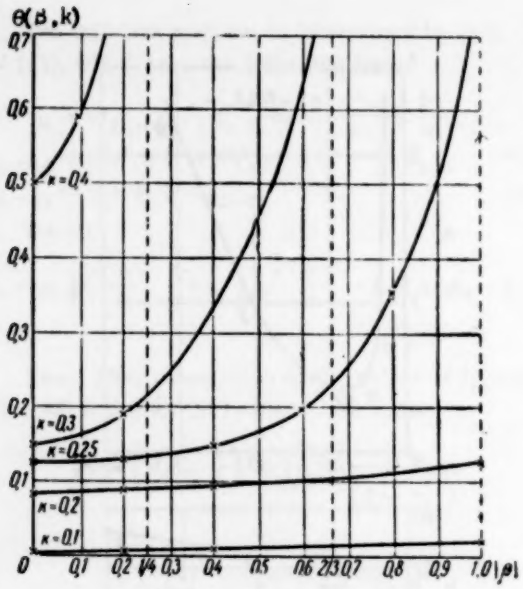

Fig. 5.

Figure 5 shows the function $\theta(\beta, k)$. It is clear from the graphs that the error from the random noise, generally speaking, increases with increasing $|\beta|$. Indeed, for $\beta = 0$ the lines $y = \text{const}$ on the (x_1, x_2) plane are concentric circles. With increasing $|\beta|$, they become ellipses whose axes are turned by 45° with respect to the coordinate axes. The greater $|\beta|$ is, the more "elongated" these ellipses become, which worsens the conditions for minimum search and increases the steady-state error from the random noise.

For $\alpha \to 0$ and $\alpha_0 \to 0$, the total steady-state error can be made arbitrarily small. Certainly, if $\gamma_1 \neq 0$, there exist optimal values of α and α_0 which are greater than zero. The essential dependence of the error on α_0 and α (or k) (Fig. 5) shows that automatic tuning of α_0 and α, providing a minimum of the error, is advantageous in automatic optimizers [5].

The author wishes to express his gratitude to B. M. Levitan for his counsel in carrying out this work.

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A STATISTICAL METHOD FOR DETERMINING DYNAMIC CHARACTERISTICS OF INDUSTRIAL OBJECTS IN THE PRESENCE OF NOISE AND FOR ANALYZING INFRALOW-FREQUENCY RANDOM PROCESSES

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The paper deals with a computer for the computation of estimates of certain statistical characteristics of stationary random functions and of linear objects' characteristics.

In the experimental investigation of the dynamic characteristics of controlled industrial objects by the method of determining an object's reaction to standard stimuli, the presence of "noise" limits, to a significant extent, the accuracy of the estimates of the high-frequency characteristics.

The investigated object itself is frequently a source of "noise." In a number of cases (for example, in the chemical industry), the "noise" level is so high, and the requirements on the technological regime so rigid, that the investigation of the dynamics of such an object by means of sufficiently large standard stimuli becomes impossible.

Today, one of the methods of investigating the dynamics of industrial objects under their actual working conditions is a statistical method, based on the correlation analysis of realizations of the input and output quantities, with the condition that these quantities, as well as the investigated object's dynamic characteristics, are stationary. The basic difficulty in analyzing industrial systems by statistical methods is that these methods require a large expenditure of labor in carrying out the computational work, in particular, in computing correlation functions or spectral densities.

With the aim of decreasing the volume of computational work, special instruments for statistical analysis were developed, both in the USSR and abroad. The instruments developed can be divided into two groups on the basis of their method of recording results:

a) recording on magnetic tape,

b) recording on paper.

There are a number of special features in the technical characteristics of instruments of the first and the second type. Instruments that record on magnetic tape are superior to instruments that record on paper as regards speed of processing statistical data; but under commutation conditions, or in the presence of magnetic fields, they are surpassed by the latter in reliability and in simplicity of use. Moreover, instruments that record on magnetic tape are designed, as a rule, for the processing of comparatively high-frequency random processes. From this point of view, it might sometimes be more convenient, in investigating the dynamics of industrial objects, to use records obtained on ruled (graph) paper.

Computers of such type, developed in the USSR and the Amsler company, were constructed on mechanical principles. Multiplication and integration in these instruments are implemented mechanically with elements which complicate preparation and increase the instrument's cost. Moreover, the existing instruments which record on paper are designed for narrowly circumscribed goals, i.e., for the computing of correlation functions, while the problem of investigating the dynamics of industrial objects requires a more complete statistical analysis.

Below, we describe an instrument for the statistical investigation of the dynamic characteristics of objects in the presence of noise, and for the analysis of infralow-frequency random processes. Considered also are the possibilities of using such an instrument for estimating statistical characteristics, weight-function parameters and for the determination of objects' frequency characteristics by statistical methods.

Instrument Capabilities and Their Uses

The instrument permits an operation of the following form to be applied to the functions $x(t)$ and $y(t)$:

$$R_{yx}(\tau, T) = \frac{1}{T}\int_0^T x(t)\, y(t-\tau)\, dt. \qquad (1)$$

1. Computing Estimates of Correlation Functions and Dispersions

Expression (1) is an estimate of the cross-correlation function of the random processes $X(t)$ and $Y(t)$. Thus, the instrument allows one to obtain estimates of cross-correlation functions. This is an unbiased estimate which converges in the mean to the true correlation function as $T \to \infty$.

If one sets $X(t) = Y(t)$, then (1) gives an estimate of the autocorrelation function of the process $X(t)$ and, in particular, for $\tau = 0$ we obtain an estimate of the dispersion.

2. Computing Coefficients of Fourier Series

If it is necessary to compute the coefficients of the Fourier series for an arbitrary function $x(t)$, then one should choose, in (1), $y(t) = \cos \omega_k t$ and $\tau = 0$. For this, a reali-

zation of x(t) in the given interval (0,T) is written on the graph paper. The realization is reproduced in the form of a voltage on potentiometer brushes, supplied from a sine-wave generator which produced voltages of the form $\sin(2\pi k/T)$ ($k = 1,2, \ldots$).

3. Computing Estimates of Spectral-Density Functions

If x(t) is a realization of the stationary random process X(t), then the value of the spectral density at the point ω_k is computed on the basis of (1)

$$M[a_k^2] = \frac{2}{T}\int_0^T R_{xx}(\tau)\cos\omega_k\tau\,d\tau + 0\left(\frac{1}{T^2}\right) \quad (2)$$

$$a_k = \frac{2}{T}\int_0^T x(t)\cos\omega_k t\,dt. \quad (3)$$

If the interval (0,T) is sufficiently large, then

$$M[a_k^2] = \frac{\pi}{T}G_{xx}(\omega_k) + 0\left(\frac{1}{T^2}\right), \quad (4)$$

where $G_{XX}(\omega_k)$ is the spectral density of the random process X(t).

The value of the dispersion $M[a_k^2]$ is computed approximately, starting from a finite number of realizations of the process X(t).

4. Determination of Frequency Characteristics by Statistical Methods

Even with a significant noise level, statistical methods permit one to find an estimate of the frequency characteristics of an object at whose input a harmonic oscillation of known frequency and amplitude is applied. Such a method is based on the following relationships.

Let there be three signals:

$$f_1(t) = D\cos\omega t,$$

$$f_2(t) = B\sin(\omega t + \theta) + n(t)$$

and

$$f_3(t) = D\sin\omega t,$$

where $f_1(t)$ is the signal at the object's input, $f_2(t)$ is the signal at the object's output which includes the reaction $B\sin(\omega t + \theta)$ to stimulus $f_1(t)$ and the "noise" n(t), with the condition that $M[n(t)] = 0$.

It is then easily verified that

$$R_{3,2}(0) = \lim_{T\to\infty}\frac{1}{T}\int_0^T f_3(t)\,f_2(t)\,dt = \frac{1}{2}BD\cos\theta, \quad (5)$$

$$R_{1,2}(0) = \lim_{T\to\infty}\frac{1}{T}\int_0^T f_1(t)\,f_2(t)\,dt = \frac{1}{2}BD\sin\theta. \quad (6)$$

On the basis of (5) and (6) we get

$$A = \frac{2}{D^2}\sqrt{R_{1,2}^2(0) + R_{3,2}^2(0)},\quad \theta = \arctan\frac{R_{1,2}(0)}{R_{3,2}(0)}, \quad (7)$$

where A = B/D is the value of the amplitude characteristic at frequency ω, and θ is the value of the phase characteristic at the same frequency.

It is easily seen that $R_{3,2}(0)$ and $R_{1,2}(0)$ are estimates of the correlation functions for $\tau = 0$. These estimates are increasingly accurate, the larger the time of integration T.

It is essential to mention that the computation of the integrals must be carried out for the function $f_2(t)$ for $t \geq t_1$, where t_1 is the time after which the natural motions of the system will, for all practical purposes, have ceased.

The instrument described in this paper permits one to execute the enumerated mathematical operations on the estimates of frequency characteristics. A scheme for taking off frequency characteristics by the statistical method is shown in Fig. 1.

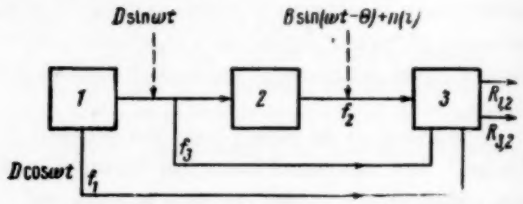

Fig. 1. 1 is a sine and cosine generator; 2 is the object; 3 is the computer.

The signal from the servosystem, which continuously reads the realization $f_2(t)$ of the random process under investigation, is applied to potentiometers which are supplied from the sine-cosine generator. The quantities thus obtained, $f_2(t)\sin\omega t$ and $f_2(t)\cos\omega t$, are applied to integrating motors. The indications of the counters on the motors' shafts correspond to the quantities $R_{3,2}(0)$ and $R_{1,2}(0)$.

5. Estimating the Parameters of Linear Objects' Weight Functions

Sometimes, to simplify a problem, one takes a rough general form of an object's transfer function; it is then necessary to determine its coefficients. As previously, we assume that there is "noise" in the object, and, consequently, we can only determine estimates of the desired parameters on the basis of recordings of the signals at the input and output of the object being investigated.

We shall consider here a method of estimating the moments of the object's weight function by means of the instrument described, for the case of objects with monotonic transient responses* (the reactions of such objects to unit disturbances are monotonic functions of time).

Industrial objects with monotonic transient responses occur quite frequently. In order to describe, in the first

*The method considered here can also be extended to the case of nonmonotonic transient responses.

approximation, the weight function of such an object, one need estimate only several parameters.

Let k(t) be the object's weight function. By the zeroth-order moment of the function k(t) we mean the number

$$\alpha_0 = \int_{-\infty}^{+\infty} k(t)\,dt. \qquad (8)$$

Since $\alpha_0 = \Phi(j\omega)|_{\omega=0}$, where $\Phi(j\omega)$ is the object's frequency characteristic, α_0 is then the static gain.

By the nth-order moment we mean the number

$$\alpha_n = \int_{-\infty}^{+\infty} t^n k(t)\,dt \qquad (n = 0,1,\ldots). \qquad (9)$$

For objects with monotonic transient responses, the weight function is completely determined, under sufficiently general conditions, by giving its moments. We note that for such objects the weight function obviously possesses the properties of a probability-density function, and the frequency characteristic possesses the properties of a characteristic function. For a complete analogy, we should introduce the normalized weight function

$$\bar{k}(t) = \frac{1}{\alpha_0} k(t) \qquad (10)$$

and, correspondingly,

$$\bar{\alpha}_n = \int_{-\infty}^{+\infty} t^n \bar{k}(t)\,dt \qquad (n = 0, 1, 2,\ldots). \qquad (11)$$

Then, the analogs of the mathematical expectation and the dispersion will be the quantities

$$\bar{\alpha}_1 = \frac{\alpha_1}{\alpha_0} \qquad (12)$$

and

$$\bar{\alpha}_{20} = \frac{\alpha_2}{\alpha_0} - \left(\frac{\alpha_1}{\alpha_0}\right)^2 = \bar{\alpha}_2 - (\bar{\alpha}_1)^2. \qquad (13)$$

The quantities α_0, $\bar{\alpha}_1$ and $\bar{\alpha}_{20}$ turn out to be very useful in the investigation of objects, and these quantities are frequently sufficient for the description, in a first approximation, of an object's functioning. These quantities have the following physical meanings: α_0 is the object's static gain; $\bar{\alpha}_1$ is the average lag of the transient response; $\bar{\alpha}_{20}$ characteristizes the distortion of the input signal.

If the object possesses a pure lag, and does not distort the form of its input signals, its weight function then obviously has the form:

$$k(t) = \delta(t - T), \qquad (14)$$

where T is the lag and $\delta(t)$ is a Dirac delta function.

Then, on the basis of (8), (11), and (13), we obtain $\alpha_0 = 1$, $\alpha_1 = \bar{\alpha}_1 = T$, and $\bar{\alpha}_{20} = 0$.

For an object with the weight function

$$k(t) = \frac{1}{T_1} e^{-(t-T)\frac{1}{T_1}} \qquad (15)$$

we have

$$\alpha_0 = 1, \bar{\alpha}_1 = T + T_1 \text{ and } \bar{\alpha}_{20} = T^2 + 2TT_1 + 2T_1^2. \qquad (16)$$

This weight function corresponds to an object consisting of a series connection of a pure delay link and an aperiodic link. We note that, on the basis of (16), we can easily compute the time constant T_1 and the true lag T of the object as functions of the average lag $\bar{\alpha}_1$ and of $\bar{\alpha}_{20}$.

Now, let k(t) be the weight function of an object consisting of two series-connected links with weight functions $k_1(t)$ and $k_2(t)$.

Then,

$$k(t) = \int_{-\infty}^{+\infty} k_1(t-\tau)\,k_2(\tau)\,d\tau, \qquad (17)$$

and for the transfer functions

$$\Phi(s) = \Phi_1(s)\,\Phi_2(s), \qquad (18)$$

where

$$\Phi_i(s) = \int_{-\infty}^{+\infty} e^{-st} k_i(t)\,dt \qquad (i = 1,2),$$

we have

$$\Phi(s) = \int_{-\infty}^{+\infty} \left[1 - st + \frac{s^2t^2}{2!} - \ldots + \right] k(t)\,dt = \alpha_0 - \alpha_1 s + \frac{\alpha_2 s^2}{2!} - \ldots + . \qquad (19)$$

On the basis of (18) and (19) we have, by equating coefficients of like powers of s,

$$\begin{aligned} \alpha_0 &= \gamma_0\beta_0, \\ \alpha_1 &= \gamma_0\beta_1 + \gamma_1\beta_0, \\ \alpha_2 &= \gamma_0\beta_2 + 2\gamma_1\beta_1 + \gamma_2\beta_0, \end{aligned} \qquad (20)$$

from which it follows that

$$\begin{aligned} \bar{\alpha}_1 &= \bar{\gamma}_1 = \bar{\beta}_1, \\ \bar{\alpha}_{20} &= \bar{\gamma}_{20} + \bar{\beta}_{20}, \end{aligned} \qquad (21)$$

where γ_n and β_n are the moments of the functions $k_1(t)$ and $k_2(t)$, and $\bar{\gamma}_n$ and $\bar{\beta}_n$ are the moments of the normalized functions $\bar{k}_1(t)$ and $\bar{k}_2(t)$ [Cf. (10)].

We now consider the expression

$$R_{yx}(t) = \int_0^T k(t)\,R_{yy}(t-\tau)\,d\tau. \qquad (22)$$

We assume here that T in (22) is chosen sufficiently large. By analogy with (20), we can write

$$\begin{aligned} a_0 &= \alpha_0 b_0, \\ a_1 &= b_0\alpha_1 + b_1\alpha_0, \\ a_2 &= b_0\alpha_2 + 2b_1\alpha_1 + b_2\alpha_2, \end{aligned} \qquad (23)$$

where

$$a_n = \int_{-\infty}^{+\infty} \tau^n R_{yx}(\tau)\, d\tau \qquad (24)$$

and

$$b_n = \int_{-\infty}^{+\infty} \tau^n R_{yy}(\tau)\, d\tau = 2\int_{0}^{\infty} \tau^n R_{yy}(\tau)\, d\tau \; (n = 0, 2, 4, \ldots). \qquad (25)$$

Since $R_{yy}(\tau)$ is an even function, we have

$$a_1 = b_0\alpha_1,$$
$$a_2 = b_0\alpha_2 + b_2\alpha_0. \qquad (26)$$

Thus, the problem of estimating the α_n reduces to estimating the moments a_n and b_n of the corresponding correlation functions, and to the solution of the system of algebraic equations (26).

The instrument considered below allows the moments a_n and b_n to be estimated. We consider the estimate of the a_n.

On the basis of (24), we can write, starting from the record of a realization on the interval $(t-T_2, t)$,

$$a_n^* = \frac{1}{T_2}\int_{-b}^{a} \tau^n\, d\tau \int_{t-T_2}^{t} x(u)\, y(u-\tau)\, du. \qquad (27)$$

The inaccuracy of the estimate in (27) is related to two circumstances: first, the infinite limits in (24) are replaced by finite ones in (27) and, secondly, instead of the cross-correlation function $R_{yx}(\tau)$, we used its estimate, obtained by an averaging operation on the finite interval $(t-T_2, t)$.

It is necessary to bear in mind that the moment of time t in (27), at which one can begin averaging to obtain the estimate of the correlation function, must be chosen subject to the condition that $t_0 \geq T$, where T is the object's "memory" (duration of the transient response). With this, t = 0 is taken as the moment when the system begins to operate.

By reversing the order of integration in (27), we get

$$a_n^* = \frac{1}{T_2}\int_{t-T_2}^{t} x(u)\, du \int_{-b}^{a} (\tau)^n y(u-\tau)\, d\tau =$$
$$= \int_{t-T_2}^{t} \left[x(u)\int_{0}^{a} (\tau)^n y(u-\tau)\, d\tau + \right.$$
$$\left. + y(u)\int_{0}^{b} (-\tau)^n x(u-\tau)\, d\tau\right] du. \qquad (28)$$

An estimate in the form of (28) can be computed in the following way. The realization y(t), written on one strip of paper, is read off in the form of a voltage and applied to a model, where one has chosen a system with finite memory a [with weight function $(\tau)^n$]. The voltage at the model's output is applied to a potentiometer whose moving contact is moved in accordance with the law x(t). The voltage at the potentiometer's output is applied to an integrating motor. The number of rotation of the motor gives the first term in (28); the second term is computed analogously.

The functional block diagram for the instrument which computes these estimates is shown in Fig. 2. For estimating the moments of the correlation function in (28), it is necessary to set

$$x(t) = y(t),$$

and then

$$b_n^* = \frac{2}{T_2}\int_{t-T_2}^{t} y(u)\, du \int_{0}^{a} \tau^n y(u-\tau)\, d\tau \; (n = 0, 2, 4, \ldots).$$

The conditions for choosing a, b, and T_2 such that the estimates of the moments a_n and b_n will be sufficient

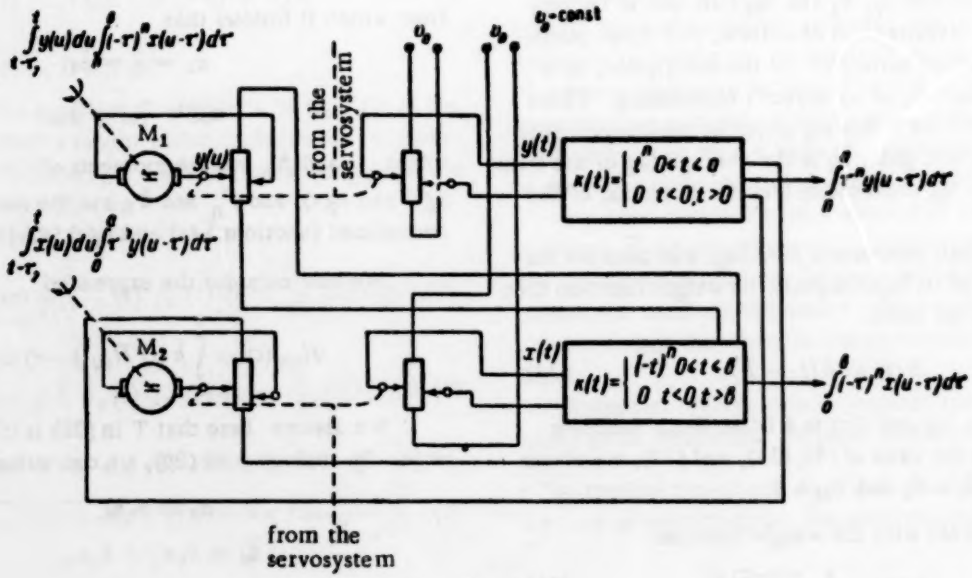

Fig. 2.

ly accurate are easily obtained if the functions $R_{xy}(t)$ and $R_{xx}(\tau)$ are known [1]. A more detailed discussion of the questions involved in the choice of a, b, and T_2 may be found in [5].

Description of the Instrument

The instrument's functional-block schematic is shown in Fig. 3, and its outside view is shown in Fig. 4. The instrument is constructed physically of two blocks. In the first block are mounted the servosystem (reading), the tape-handling mechanism, and the multiplication potentiometer.

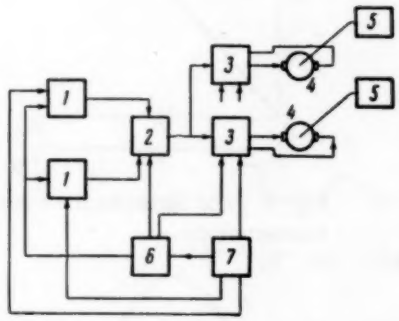

Fig. 3. 1 is a reading device (servosystem); 2 is a multiplier; 3 is a power amplifier; 4 is an integrating motor, 5 is a rotation counter; 6 is a stabilized power supply and 7 is a control panel.

The operation of the servosystem is based on the principle of tracing by a photohead along the edge of a curve drawn on the graph paper. (By the edge of a curve we mean the boundary between the black and white regions.) The photohead consists of a photoimpedance, which is the sensitive element, and a small light bulb.

The photoimpedance is placed on the front side, and the light bulb on the internal side, of the graph-paper strip. The photoimpedance is connected in one of the arms of the servosystem's bridge circuit. The bridge circuit is supplied from a transformer.

Depending on whether the illuminated area of the photoimpedance is increased or decreased, its impedance will decrease or increase, causing a signal of the proper phase to be applied to the input of the phase-sensitive amplifier, made up of twin triodes in a bridge circuit.

The servosystem is executed on the basis of programing device RU5-01 (02). The block schematic of the servosystem's mock-up was changed, permitting the instrument's dynamic characteristics to be increased, i.e., the servosystem's processing speed was increased from 12 to 100 mm/sec.

Depending on the phase of the input signal to a reversible two-phase induction motor, whose control winding is connected in the amplifier's cathode circuit and which is turned in the proper direction, the carriage on which the photohead is mounted will move so as to decrease the imbalance in the bridge circuit.

At the moment when the program's edge covers that portion of the light-sensitive layer of the photoimpedance for which the bridge circuit is balanced, the error signal will equal zero.

Synchronous motors serve as the tape-handling mechanism of the instrument. A realization is carried separately on two reels of graph paper. Reading is implemented separately by the two reading servosystems. Such a kinematic scheme allows the lag τ to be established comparatively simply and within broad limits. The

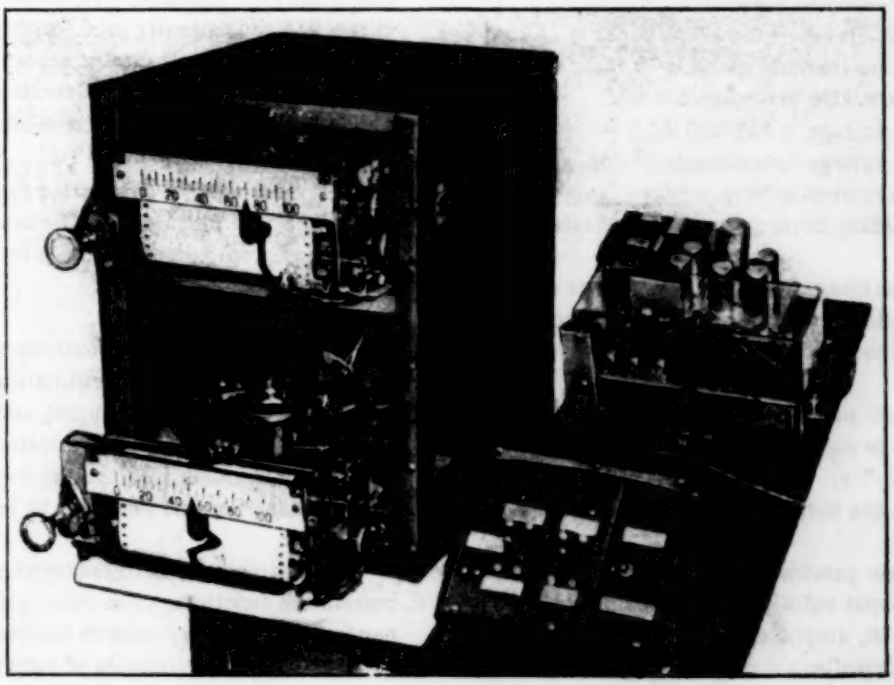

Fig. 4

speeds of the tape-handling drives are synchronized. Multiplication is affected by potentiometers, of which there are two pairs. The moving contacts of the paired potentiometers are rigidly connected to each other.

The resistance ratio of the multiplication potentiometers was designed to be 1:20, for the purpose of avoiding the distortions of their characteristics caused by the loading of one potentiometer by the other. The potentiometers are made up of bridge circuits for the compensation of the voltages corresponding to the mathematical expectations of the random functions.

In this block, the mechanism for reversing the tape-handling drives is also mounted. The second block contains:

1) two ferroresonant voltage stabilizers with two half-period rectifiers and filters;

2) two semiconductor stabilized-voltage sources based on Zener pedestal diodes, used for supplying the multiplication potentiometers;

3) two semiconductor phase-sensitive power amplifiers for controlling the integrating motors, whose inputs are the multiplication potentiometers' output circuits;

4) two integrating motors with rotation counters;

5) the control panel.

The control panel is used for starting, reversing, and stopping the instrument. Instrument stopping is also effected by limit switches on the tape-handling mechanisms. When the limit switches operate, the tape-handling mechanism and integrating motors, whose windings are shorted, are disconnected. The limit switches operate when an index from the pencil on the graph paper hits the transducer of the limit switch. Execution of the various computational tasks on the instrument is implemented by various combinations of its elements.

Technical Characteristics

1. Speed of travel of the graph paper is 15 mm/sec.
2. Maximum tracking speed is 10 mm/sec.
3. Static-tracking accuracy is 0.5%.
4. Supply voltage is 127/220 v.
5. Supply-voltage frequency is 50 cps.
6. Power required is 30 w.
7. The reading device's scale is graduated from 0 to 100 %.
8. The maximum length of graph paper is 16 m.
9. The basic parameters of the ferroresonant voltage stabilizer are: input voltage 12 v, and output voltage 22/25 v.
10. The basic parameters of the semiconductor stabilized-voltage supply are: input voltage 22 v, and output voltage 8.7 v. With variations of the input voltage by ±20%, the output voltage remains virtually unchanged.
11. The basic parameters of the phase-sensitive power amplifier are: input voltage ± 6.7 v, output voltage ± 16 v, current gain of 10, amplifier characteristic linear to within 2%. If the amplifier's voltage supply varies from -10 to 20%, the voltage on the load remains virtually unaltered.

Experimental Characteristics

Figure 5 shows the curve for the number n of rotations of the integrating motor during 30 seconds as a function of the voltages u_2 and u_1 at the inputs of the amplifier and motor respectively.

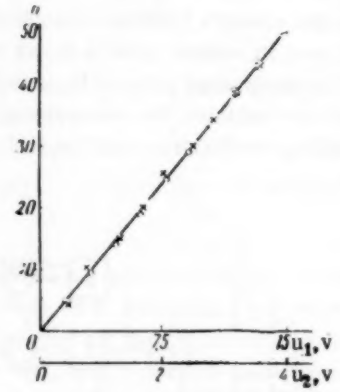

Fig. 5. The instrument's integral characteristic.

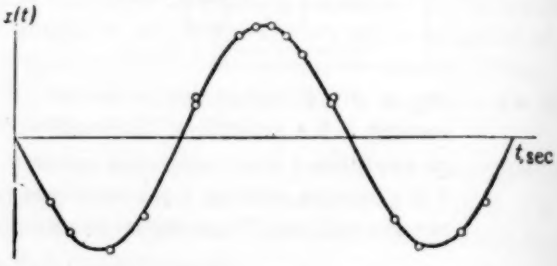

Fig. 6

Figure 6 shows the results of applying operator (1) to the function $\sin(\omega t + \theta)$, it being known that the form of this curve is invariant (as $T \to \infty$) with respect to transformation (1). Consequently, sinusoidal (or cosinusoidal) curves are conveniently used for verifying the instrument's accuracy. The small circles denote the ordinates of the curve computed by means of the instrument. As is clear, the deviation of the experimental function from the computed one does not exceed 4%.

Figure 7 shows a correlation function computed with the help of the instrument. The realization of the corresponding random process is shown in Fig. 8.

Conclusions

This work considered methods of analyzing low-frequency random processes, estimating the parameters of objects in the presence of noise, and also described an instrument for solving such problems.

The instrument's computing device permits the following quantities and functions to be computed automatically:

1) estimates of mathematical expectations, autocorrelation functions, cross-correlation functions and dispersions of stationary random functions;

2) Fourier coefficients of individual realizations;

3) estimates of spectral-density functions;

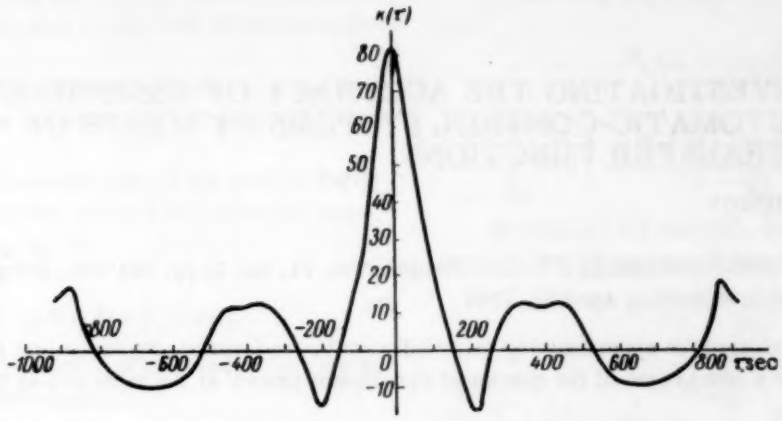

Fig. 7

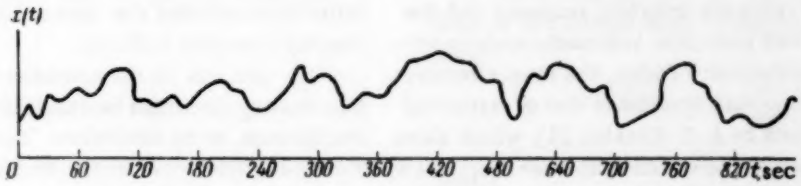

Fig. 8

4) estimates of systems' frequency characteristics;

5) estimates of weight-function moments.

The instrument developed is very simple to use, and is very reliable. These two qualities make it particularly convenient for use in the analysis of industrial objects, and also nicely distinguish it from electronic computing devices with complex circuitry and low reliability.

Among the superior features of this instrument is its capability of using realizations written on paper, since widely used industrial apparatus permits recording on paper tape.

Among the disadvantages of the instrument is the large amount of time necessary for an analysis.

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METHOD OF INVESTIGATING THE ACCURACY OF ESSENTIALLY NONLINEAR AUTOMATIC-CONTROL SYSTEMS BY MEANS OF EQUIVALENT-TRANSFER FUNCTIONS

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A method is considered for approximating essentially nonlinear functions by equivalent frequency characteristics on the basis of a comparison of the spectra of the random process at the input and at the output of the nonlinear element.

One of the complicated problems in automatic-control theory is that of analyzing the dynamic accuracy and the forced motions of closed nonlinear automatic systems acted on by random disturbances. Today, the most effective method for investigating such systems is that of statistical linearization, developed by I. E. Kazakov [1], which starts from the representation of inertialess nonlinear elements by constant equivalent coefficients. This coefficient is most simply defined by one or another of two assumptions and, with this, two values are obtained for the coefficient.

Definition of the coefficient by the condition that the dispersions at the output of the true nonlinear element and of the equivalent linear one be equal leads to the expression

$$k_1^{(1)} = \frac{\sigma_y}{\sigma_x}, \qquad (1)$$

where σ_x is the mean-square deviation of the random function at the input of the nonlinear element and σ_y is the mean-square deviation at its output.

The definition of the coefficient starting from the requirement that the mean-square difference between the true and the approximating random functions be a minimum leads to an expression of the form

$$k_1^{(2)} = \frac{R_{yx}(\tau v)}{\sigma_x^2}, \qquad (2)$$

where $R_{yx}(\tau)$ is the cross-correlation function of the random processes at the nonlinear element's input and output.

In both cases, the nonlinear element's transfer factor by average value is defined as the ratio of the mathematical expectations at the output and input of this element:

$$k_0 = \frac{m_y}{m_x}, \qquad (3)$$

where m_x and m_y are the mathematical expectations of the random processes at the nonlinear element's input and output respectively.

Approximation by the coefficients in (1) and (2) gives good results with the condition that a narrow-band filter stands beyond the nonlinear element. These coefficients are conveniently used only in the analysis of the operational accuracy of simple automatic-control systems which do not contain complicated filters in their structure since, in this case, the representation of the true spectral content of the random process in the nonlinear system is lost. This latter circumstance also makes the synthesis of optimal nonlinear systems difficult.

We turn now to the consideration of a method of approximating nonlinear functions by equivalent frequency coefficients, or by equivalent "transfer" functions, which more completely mirror the true outlines of the random process's passage through the nonlinearity.

This method allows one to compute, with as great an accuracy as desired, the dispersion of any coordinate of a nonlinear system, and opens up a path for the synthesis of optimal filters.

An equivalent frequency-transfer factor for a nonlinear element is defined as the square root of the ratio of the spectral densities of the random process at the input and output:

$$K(\omega, \sigma_x, m_x) = \sqrt{\frac{S_{out}(\omega)}{S_{in}(\omega)}}, \qquad (4)$$

where $S_{in}(\omega)$ is the spectral density of the random process at the nonlinear element's input, and $S_{out}(\omega)$ is that at its output.

Naturally, $K(\omega, \sigma_x, m_x)$ is also a function of σ_x and m_x, and depends on the type of nonlinear characteristic.

For the determination of the frequency characteristic, it is necessary to know the spectral density of the process at the output. We assume that we know the two-dimensional-distribution law and the correlation function of the random process at the input of the nonlinear element, where the distribution law is a normal one.

For a stationary random process, the correlation function (the autocorrelation function) is defined by the expression

$$R_x(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 - m\,)(x_2 - m_x) W_2(x_1\ x_2, \tau)\, dx_1, dx_2,$$

where $W_2(x_1, x_2, \tau)$ is the random process's two-dimensional probability density.

We now consider the nonlinear functional dependency

$$y = f(x). \qquad (6)$$

The autocorrelation function $R_y(\tau)$ at the nonlinear

element's output will be the average value of the product $y(t)y(t+\tau)$, which depends on the form of nonlinear function (6).

If the values of the input signal equal $x(t)$ and $x(t+\tau)$, then the values of the output signal will be $f[x(t)]$ and $f[x(t+\tau)]$.

Consequently, the probability of the product $f[x(t)]\, f[x(t+\tau)]$ will determine the two -dimensional probability density $W_2(x_1, x_2, \tau)$.

The mean value

$$M\{f[x(t)]\, f[x(t+\tau)]\} \tag{7}$$

is the autocorrelation function of the random process at the nonlinear element's output.

Expression (7) can be written in the alternative form

$$R_y(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x_1)\, f(x_2)\, W_2(x_1, x_2, \tau)\, dx_1\, dx_2, \tag{8}$$

where

$$x_1 = x(t),\quad x_2 = x(t+\tau).$$

In (8), the random process is taken to have zero mean value.

For the computation of the integral in (8), we approximate the nonlinear function of (6) by rectilinear segments and, for $W_2(x_1, x_2, \tau)$, we use Cramer's expansion by terms of orthogonal Hermite polynomials [2].

The Cramer expansion has the form:

$$\frac{1}{\sqrt{1-\rho^2}}\exp\left[-\frac{\rho^2x_1^2+\rho^2x_2^2-2\rho x_1x_2}{2(1-\rho^2)}\right] = \sum_{\nu=0}^{\infty}\frac{H_\nu(x_1)\,H_\nu(x_2)}{\nu!}\rho^\nu, \tag{9}$$

where $\rho = \dfrac{R(\tau)}{R(0)}$ is the normalized autocorrelation function, and $H_\nu(x)$ ($\nu = 1,2,3,\ldots$) are orthogonal Hermite polynomials.

For the two-dimensional normal probability-density function, we get

$$\frac{1}{2\pi\sqrt{1-\rho^2}}\exp\left[-\frac{\rho^2x_1^2+\rho^2x_2^2-2\rho x_1x_2}{2(1-\rho^2)}\right]\times\exp\left[-\frac{x_1^2+x_2^2}{2}\right] = \exp\left[-\frac{x_1^2+x_2^2}{2}\right]\sum_{\nu=0}^{\infty}\frac{H_\nu(x_1)\,H_\nu(x_2)}{2\pi\nu!}\rho^\nu \tag{10}$$

or

$$\frac{1}{2\pi\sqrt{1-\rho^2}}\exp\left[-\frac{x_1^2+x_2^2-2\rho x_1x_2}{2(1-\rho^2)}\right] = \exp\left[-\frac{x_1^2+x_2^2}{2}\right]\sum_{\nu=0}^{\infty}\frac{H_\nu(x_1)\,H_\nu(x_2)}{2\pi\nu!}\rho^\nu. \tag{11}$$

We now normalize the random variable x with respect to the mean square value σ, i.e., we set $\xi_1 = x_1/\sigma_1$ and $\xi_2 = x_2/\sigma_2$.

In the sequel, we can take $\sigma_1 = \sigma_2 = \sigma$, since the random process is stationary.

Sometimes, the integral in (8) can be written in the form

$$R_y(\tau) = \frac{1}{2\pi\sqrt{1-\rho^2}}\times \tag{12}$$
$$\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp\left[-\frac{\xi_1^2+\xi_2^2-2\rho\xi_1\xi_2}{2(1-\rho^2)}\right] f(\sigma\xi_1)\, f(\sigma\xi_2)\, d\xi_1 d\xi_2.$$

By joining (11) and (12), we obtain the following expression for the autocorrelation function:

$$R_y(\tau) = \frac{1}{2\pi\nu!}\sum_{\nu=0}^{\infty}\rho^\nu\times$$
$$\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\sigma\xi_1)\, f(\sigma\xi_2)\, e^{-\frac{\xi_1^2+\xi_2^2}{2}}\, H_\nu(\xi_1)\, H_\nu(\xi_2)\, d\xi_1\, d\xi_2. \tag{13}$$

The double integral in (13) decomposes into the product of two simple ones. Then (13) may be given in the form:

$$R_y(\tau) = \sum_{\nu=0}^{\infty} a_\nu^2\rho^\nu, \tag{14}$$

where

$$a_\nu = \frac{1}{\sqrt{2\pi\nu!}}\int_{-\infty}^{\infty} f(\sigma\xi)\, H_\nu(\xi)\, e^{-\frac{\xi^2}{2}}\, d\xi. \tag{15}$$

The orthogonal Hermite polynomials are defined by the relationship

$$\left(\frac{d}{d\xi}\right)^\nu e^{-\frac{\xi^2}{2}} = (-1)^\nu H_\nu e^{-\frac{\xi^2}{2}} \tag{16}$$

and equal, for various values of ν, respectively:

$$H_0(\xi) = 1,\ H_1(\xi) = \xi,\ H_2(\xi) = \xi^2 - 1,$$
$$H_3(\xi) = \xi^3 - 3\xi,\ H_4(\xi) = \xi^4 - 6\xi^2 + 3, \tag{17}$$
$$H_5(\xi) = \xi^5 - 10\xi^3 - 15\xi.$$

Thus, the autocorrelation function at the nonlinear element's output can be determined by computing the series in (14). This series is absolutely convergent, since $\rho(\tau)$ is a bounded function, and the coefficients a_ν decrease at the rate $1/\nu$!.

If the nonlinear device's characteristic is an antisymmetric function, then (14) contains only odd terms of the expansion

$$R_y(\tau) = a_1^2\rho + a_3^2\rho^3 + a_5^2\rho^5 + \cdots \tag{18}$$

If this function is odd, but not symmetric, then all the terms of the series are retained. With this, the coefficient a_0^2 equals the square of the mean value at the output which results from the lack of symmetry.

In this case, the autocorrelation function for the centered random process will be defined by

$$R_y^0(\tau) = \sum_{\nu=1}^{\infty} a_\nu^2\rho^\nu. \tag{19}$$

From a known autocorrelation function, we obtain the spectral density by a Fourier transformation.

We may now turn to the definition of the equivalent frequency transfer factor. For the determination of this coefficient, it turns out that it suffices to know the a_ν and the spectral densities of the powers of the autocorrelation function at the nonlinear element's input.

Then, by the definition of (4), $K(\omega, \sigma_x, m_x)$ will have the form:

$$K(\omega, \sigma_x\ m_x) = \frac{1}{\sigma}\sqrt{\sum_{\nu=1}^{\infty} a_\nu^2 b_\nu}, \qquad (20)$$

where

$$b_\nu = \frac{S^\nu(\omega)}{S_{in}(\omega)}, \quad S^\nu(\omega) \div \rho^\nu(\tau).$$

For example, for correlation functions and spectral densities of the form

$$\rho_x(\tau) = e^{-\beta|\tau|}, \quad S_x(\omega) = \frac{2\beta}{\omega^2 + \beta^2} \qquad (21)$$

the coefficients b_ν equal:

$$\begin{aligned} b_1 &= 1, \\ b_2 &= 2\frac{\omega^2 + \beta^2}{\omega^2 + 4\beta^2} \\ b_3 &= 3\frac{\omega^2 + \beta^2}{\omega^2 + 9\beta^2}, \\ b_4 &= 4\frac{\omega^2 + \beta^2}{\omega^2 + 16\beta^2}, \\ b_5 &= 5\frac{\omega^2 + \beta^2}{\omega^2 + 25\beta^2}, \\ &\cdots\cdots\cdots\cdots \end{aligned} \qquad (22)$$

In this case, the transfer coefficient equals

$$K(\omega, \sigma_x, m_x) =$$

$$= \frac{1}{\sigma_x}\sqrt{a_1^2 + 2a_2^2\frac{\omega^2 + \beta^2}{\omega^2 + 4\beta^2} + 3a_3^2\frac{\omega^2 + \beta^2}{\omega^2 + 9\beta^2} + \cdots} \qquad (23)$$

In accordance with (14), the coefficients a_ν^2 are proportional to the dispersions, and may be computed for the standard essential nonlinearities.

In the given case, the transfer factor by mean value is defined by the expression

$$k_0(m_x, \sigma_x) = \frac{km_x - a_0}{m_x}, \qquad (24)$$

where k is the slope of the nonlinear characteristic at the working point.

We now consider (20) in more detail. If we assume that the spectral densities at the nonlinear element's input and output have the same form, then the coefficients $b_\nu = 1$. As a result, we get the transfer coefficient

$$K(\sigma_x, m_x) = \frac{1}{\sigma_x}\sqrt{\sum_{\nu=1}^{\infty} a_\nu^2} \equiv k_1^{(1)}, \qquad (25)$$

which corresponds to equivalent transfer factor (1), computed by the first approximation method, since

$$\sum_{\nu=1}^{\infty} a_\nu^2 = \sigma_y^2.$$

If we take only the first term of the series, we obtain the transfer factor

$$K(\sigma_x, m_x) = \frac{a_1}{\sigma_x} \equiv k_1^{(2)}, \qquad (26)$$

which corresponds to the equivalent coefficient in (2), computed by the second approximation method.

If we neglect all the coefficients a_ν except a_1, this means physically that we are not taking into account the components of the random process due to the nonlinear transformation.

Thus, (25) and (26) serve as the mathematical bases for the two methods of statistical linearization of nonlinear functions.

We now turn to the consideration of the equivalent "transfer" function for a nonlinear element. Taking into consideration that the equivalent frequency-transfer factor is taken as the ratio of spectral densities, which are even functions of frequency, we present it in the form of a product of complex conjugate factors:

$$K^*(j\omega, \sigma_x, m_x)K(j\omega, \sigma_x, m_x) = K^2(\omega, \sigma_x, m_x). \qquad (27)$$

By choosing the zeroes and poles in the upper half of the complex plane, we obtain the following expression for the equivalent "transfer" function:

$$K(j\omega, \sigma_x, m_x) = k_{st}\frac{(T_1 j\omega + 1)(T_3 j\omega + 1)\ldots}{(T_2 j\omega + 1)(T_4 j\omega + 1)\ldots}, \qquad (28)$$

where the value of

$$k_{st} = \frac{1}{\sigma_x}\sqrt{a_1^2 + \frac{1}{2}a_2^2 + \frac{1}{3}a_3^2 + \cdots} \qquad (29)$$

is found from (23) for $\omega = 0$; T_1, T_2, T_3,... are time constants, and are functions of the coefficients a_ν, and of the form of the spectral density at the input, for which $T_1 > T_2$, $T_3 > T_4$, etc.

The coefficient in (29) has a value lying between the values of the coefficients in (25) and (26), and can give as accurate a value as desired for the computation of the dispersion in a nonlinear system.

However, this fact holds in the case when, at the input of the nonlinear element, there acts a random process with a spectral density in the form of "white" noise, or with a frequency band which exceeds the pass band of the linear filter which follows the nonlinearity. In all other cases, it is desirable, in analyzing dynamic accuracy, to use the equivalent "transfer" function of (28).

Since (20), (28), and (29) have been obtained in the form of series, it suffices for practical computations to limit oneself to five, and sometimes even three, terms of the expansion.

It is obvious that, both for the computation of equivalent transfer factors and for the determination of the equivalent "transfer" function, it is necessary to know the coefficients a_ν, defined by the integral in (15). These coefficients are functions of the dispersion of the input random process, and depend on the form of the nonlinear function.

The computational formulas for the a_ν may be given for any nonlinear characteristic.

Let us consider the methodology of computing the a_ν. In (15) the nonlinear function $f(\sigma\xi)$ enters, complicating the computation of the integral, so that we approximate this function by rectilinear segments. Then, the integral of (15) may be easily computed by using tables of the incomplete gamma function and of the χ^2 probability function [3].

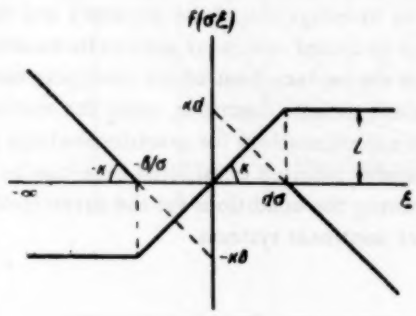


Fig. 1.

We illustrate this methodology by an example. We take a nonlinearity of the limitation type, as shown in Fig. 1, and write the general expression for the a_ν, which has the form:

$$a_\nu = \frac{k\sigma}{\sqrt{2\pi\nu!}}\int_{\frac{d}{\sigma}}^{\infty} \xi H_\nu(\xi)\, e^{-\frac{\xi^2}{2}}\, d\xi + \\ + \frac{k}{\sqrt{2\pi\nu!}}\int_{\frac{d}{\sigma}}^{\infty} (d-\sigma\xi) H_\nu(\xi)\, e^{-\frac{\xi^2}{2}}\, d\xi + \\ + \frac{k\sigma}{\sqrt{2\pi\nu!}}\int_{-\infty}^{\nu} \xi H_\nu(\xi)\, e^{-\frac{\xi^2}{2}} d\xi + \\ + \frac{k}{\sqrt{2\pi\nu!}}\int_{-\infty}^{-\frac{b}{\sigma}} (-b-\sigma\xi)\, H_\nu(\xi)\, e^{-\frac{\xi^2}{2}}\, d\xi. \qquad (30)$$

In the integrands of (30) there appear powers of the variable which depend on the order of the polynomials in (16).

These integrals may be reduced to the tabulated ones:

$$\int_0^x \xi^n e^{-\frac{\xi^2}{2}}\, d\xi = \sqrt{2}^{n-1}\left(\frac{n-1}{2}\right)!\,[1 - P(\chi^2,\ n+1)] \qquad (31)$$

or

$$\frac{1}{2^{\frac{n-2}{2}}\Gamma\left(\frac{n}{2}\right)}\int_\chi^\infty \xi^{n-1} e^{-\frac{\xi^2}{2}}\, d\xi = P(\chi^2,\ n). \qquad (32)$$

The function $P(\chi^2, n)$ is a function of two arguments for which tables have been computed (Cf. [2]).

In the given case, the quantity χ is defined as

$$\chi = \frac{d}{\sigma}, \qquad (33)$$

where

$$d = h \pm m_x.$$

For the nonlinear characteristic shown in Fig. 1,

$$\chi_1 = \frac{h - m_x}{\sigma_x} = \frac{d}{\sigma_x},$$

$$\chi_2 = \frac{-h - m_x}{\sigma_x} = -\frac{b}{\sigma_x}.$$

Then,

$$a_0 = L\left\{\frac{1}{2}\left[\left|\frac{h-m}{h}\right| P(\chi_1^2,\ 1) - \frac{h+m}{h} P(\chi_2^2,\ 1)\right] + \right. \\ \left. + \frac{1}{\sqrt{2\pi}}\frac{\sigma}{h}\left[-P(\chi_1^2,\ 2) + P(\chi_2^2,\ 2)\right]\right\}, \qquad (34)$$

$$a_1 = L\left\{\frac{\sigma}{h}\left[1 - \frac{1}{2}\left(P(\chi_1^2,\ 3) + P(\chi_2^2,\ 3)\right)\right] + \right. \\ + \frac{1}{\sqrt{2\pi}}\frac{m}{h}\left[P(\chi_2^2,\ 2) - P(\chi_1^2,\ 2)\right] + \\ \left. + \frac{1}{\sqrt{2\pi}}\left[P(\chi_1^2,\ 2) + P(\chi_2^2,\ 2)\right]\right\}$$

etc.

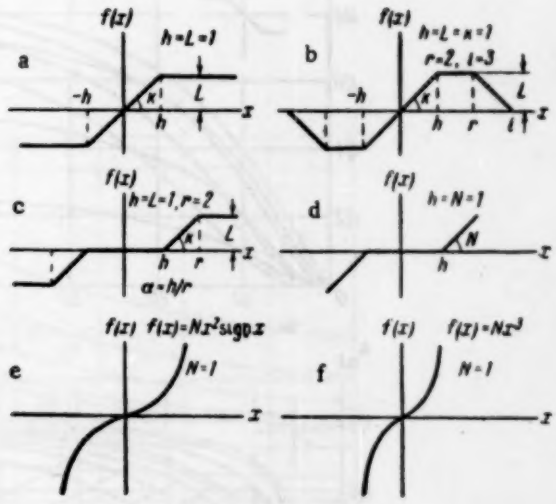


Fig. 2.

Using (34), set up for the nonlinear characteristics shown in Fig. 2, we have calculated and constructed the graphs for the normalized coefficients $a_{n\nu}$ which are given in the Appendix in Figs. 3-28. The parameter m on the graphs has the scale of the quantity x/σ.

In computing the coefficients a_ν from these graphs, it is necessary to take the following relationships into account:

1) for nonlinearities of the limitation type (Fig. 2,a and b), $a_\nu = a_{n\nu} L$

2) for dead zones with limitations (Fig. 2, c),

$$a_\nu = a_{n\nu}\frac{L}{1-\alpha}, \qquad \alpha = \frac{h}{r},$$

3) for dead zones, cubic parabolas and antisymmetric quadratic parabolas (Fig. 2,d,e and f),

$$a_\nu = a_{n\nu} N$$

Example. In conclusion, we give the example of the determination of the equivalent "transfer" function for a nonlinearity of the cubic-parabola type of Fig. 2,f.

Let.

$$N = 1, \quad S_{in}(\omega) = \frac{2\beta}{\omega^2 + \beta^2}, \quad \sigma_x = 1{,}5, \quad \beta = 10,$$

$$K(j\omega, \sigma_x, m_x) = \frac{1}{\sigma_x} \sqrt{a_1^2 + \frac{a_3^2}{3}} \frac{\sqrt{\frac{a_1^2 + 3a_3^2}{3\beta^2(3a_1^2 + a_3^2)}} j\omega + 1}{\frac{1}{3\beta} j\omega + 1} \tag{35}$$

or

$$K(s, \sigma_x, m_x) = k_{st} \frac{T_1 s + 1}{T_2 s + 1}, \qquad s = j\omega,$$

where

$$K_{st} = \frac{1}{\sigma_x} \sqrt{a_1^2 + \frac{a_3^2}{3}} = 2{,}25 \sqrt{11},$$

$$T_1 = \sqrt{\frac{a_1^2 + 3a_3^2}{3\beta^2(3a_1^2 + a_3^2)}} = 0{,}1 \sqrt{\frac{3}{11}},$$

$$T_2 = \frac{1}{3\beta} = \frac{1}{30}.$$

Then,

$$K(s, \sigma_x, m_x) = 7{,}5 \frac{0{,}0524s + 1}{0{,}0333s + 1}. \tag{36}$$

The investigation of the accuracy and the forced motions of closed nonlinear automatic-control systems leads to the replacement of the nonlinear elements by equivalent transfer functions, using the method of successive approximations for graphicoanalytic methods. The transfer function considered here can be used for determining the conditions for the investigation of stability of nonlinear systems.

APPENDIX

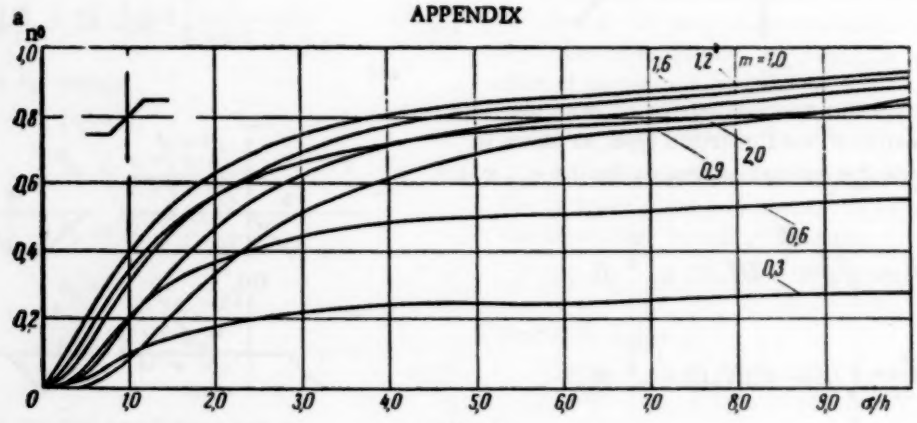


Fig. 3.

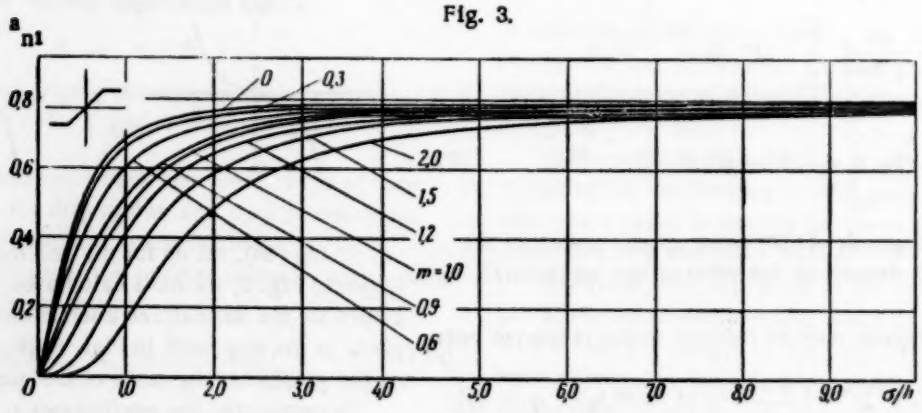


Fig. 4.

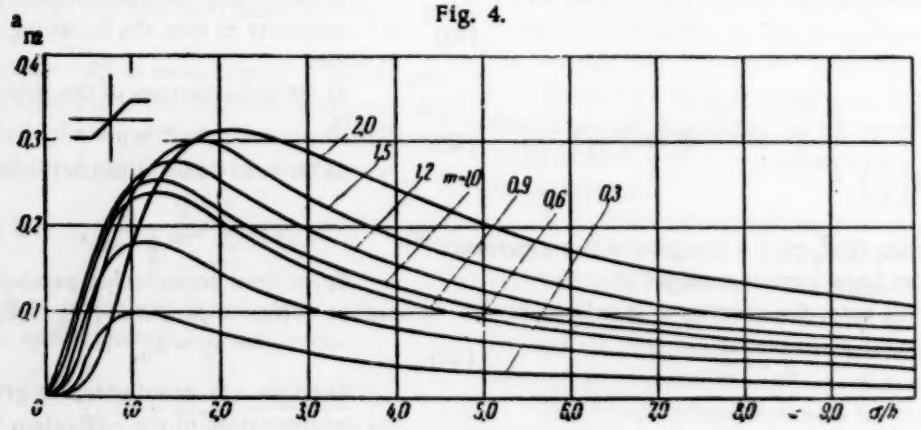


Fig. 5.

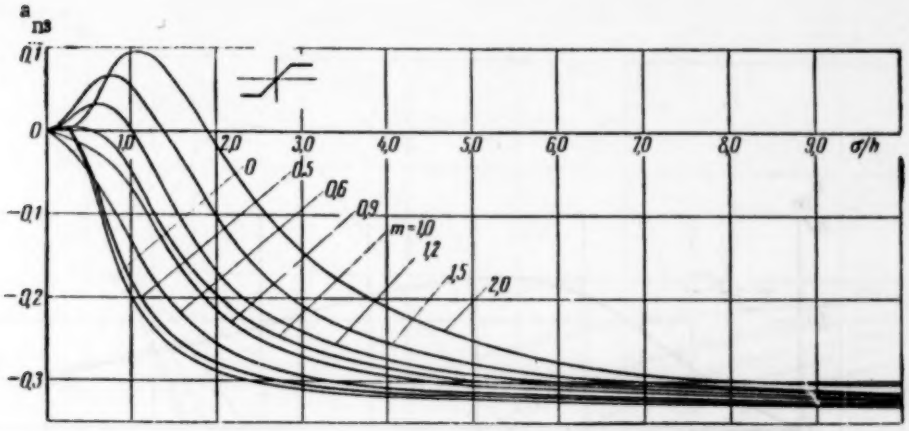
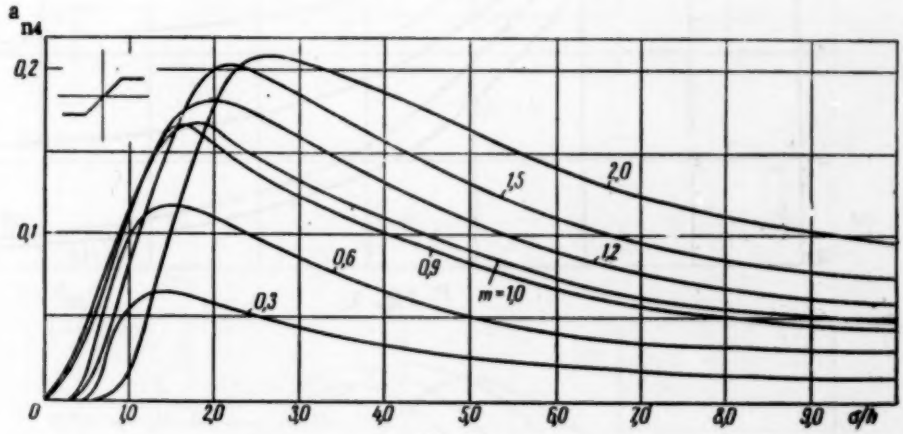


Fig. 6.



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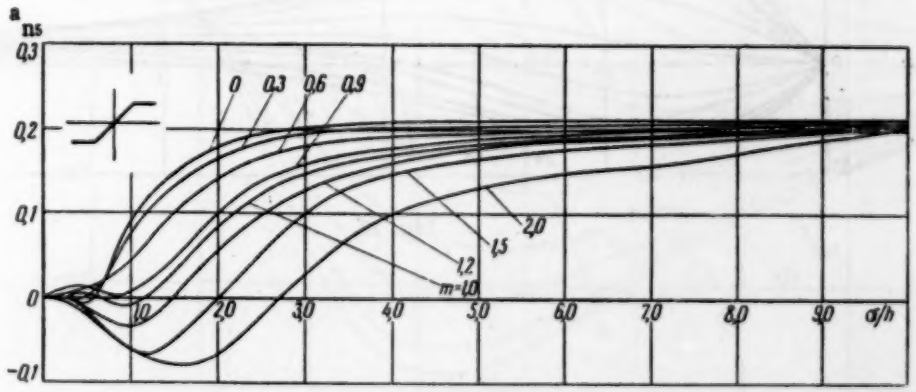
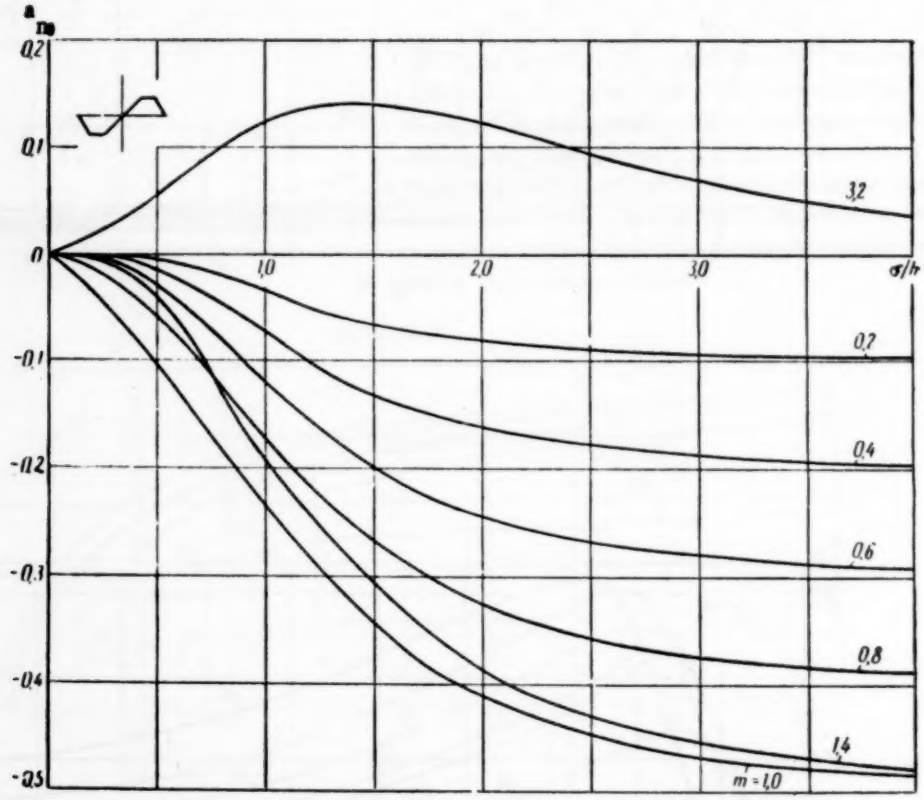


Fig. 8.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0,2
0,1
0
-0,1
-0,2
-0,3
-0,4
-0,5
1,0
2,0
3,0
σ/h
3,2
0,2
0,4
0,6
0,8
1,4
m = 1,0

Fig. 9.

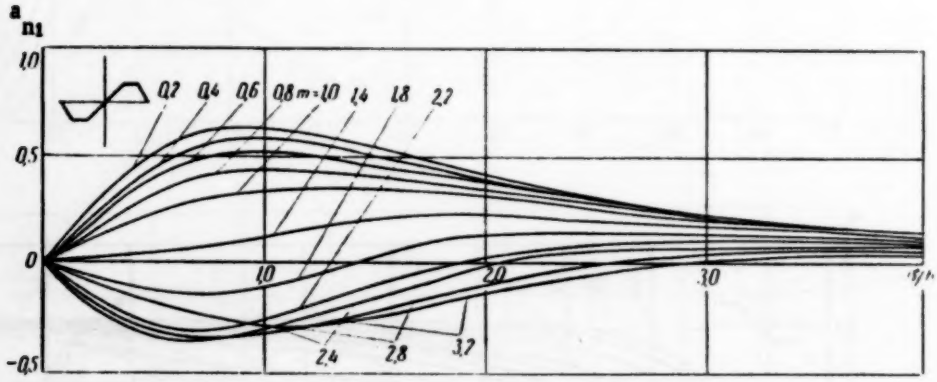

a n1
1,0
0,5
0
-0,5
0,2
0,4
0,6
0,8
m=1,0
1,4
1,8
2,2
1,0
2,0
3,0
2,4
2,8
3,2

Fig. 10.

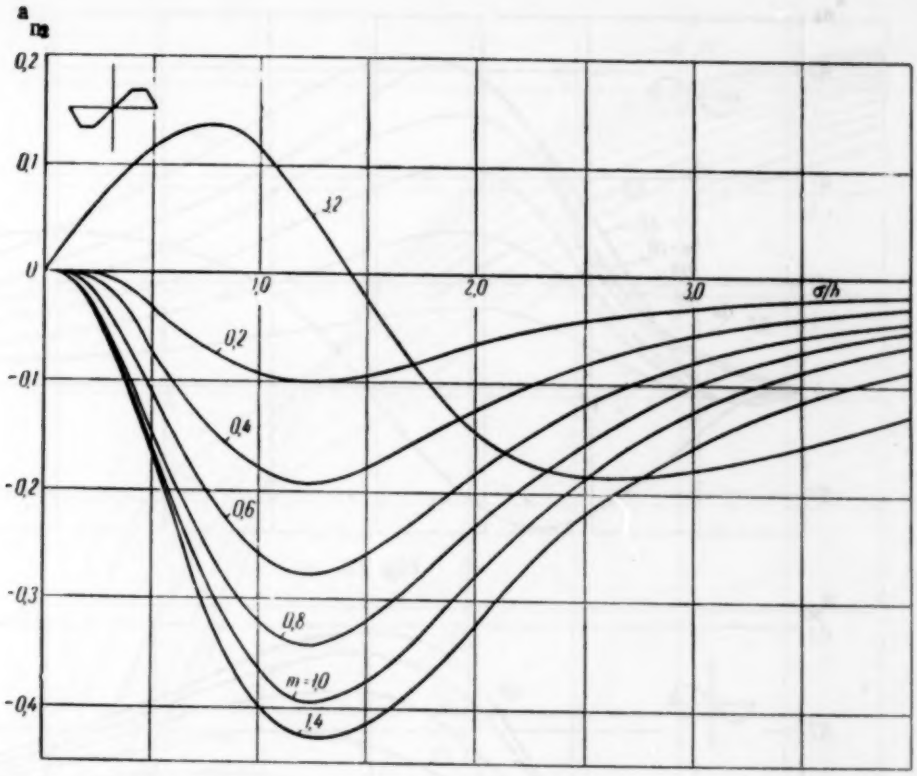

a
ns
0,2
0,1
0
-0,1
-0,2
-0,3
-0,4
1,0
2,0
3,0
σ/h
1,2
0,2
0,4
0,6
0,8
m=1,0
1,4

Fig. 11.

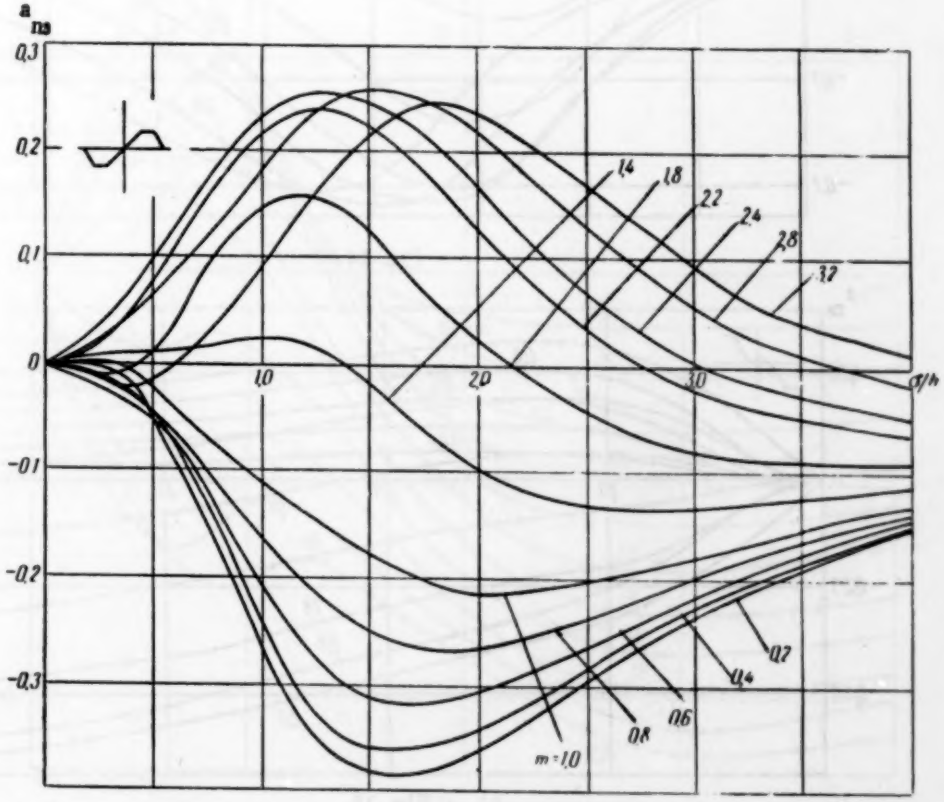
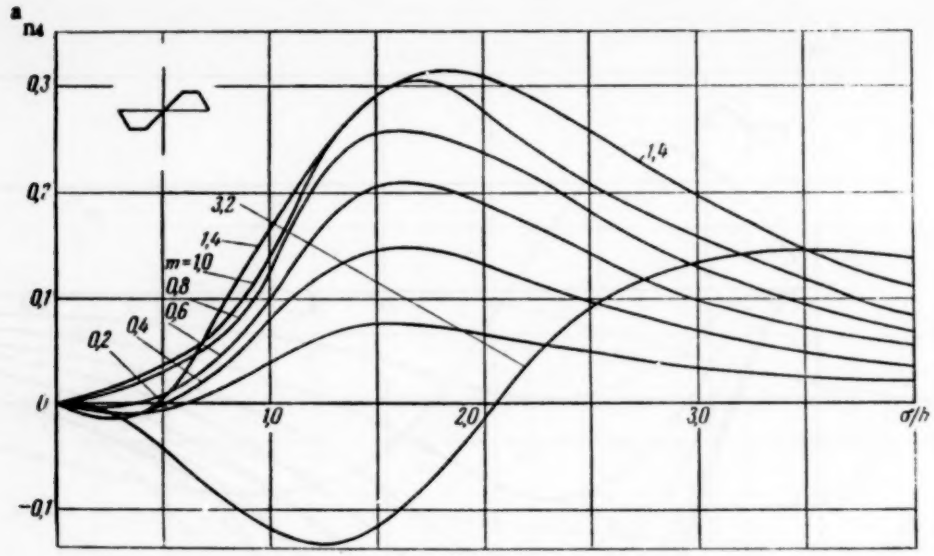

a
ns
0,3
0,2
0,1
0
-0,1
-0,2
-0,3
1,0
2,0
3,0
σ/h
1,4
1,8
2,2
2,4
2,8
3,2
0,7
0,4
0,6
0,8
m=1,0

Fig. 12.



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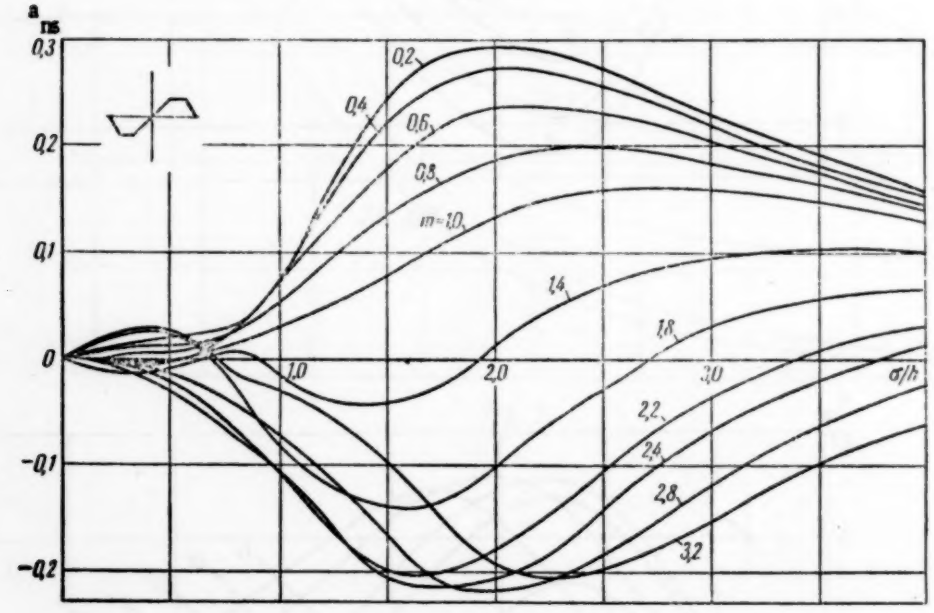


Fig. 14

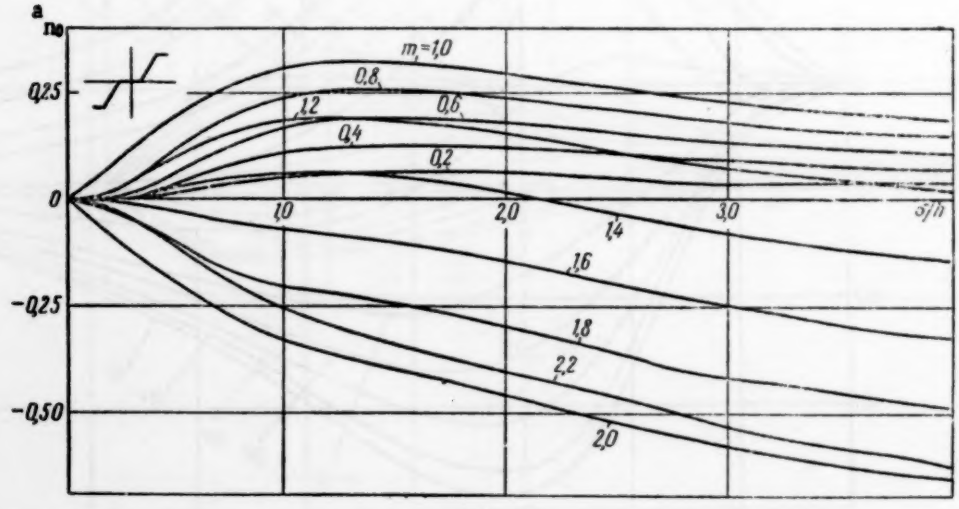


Fig. 15

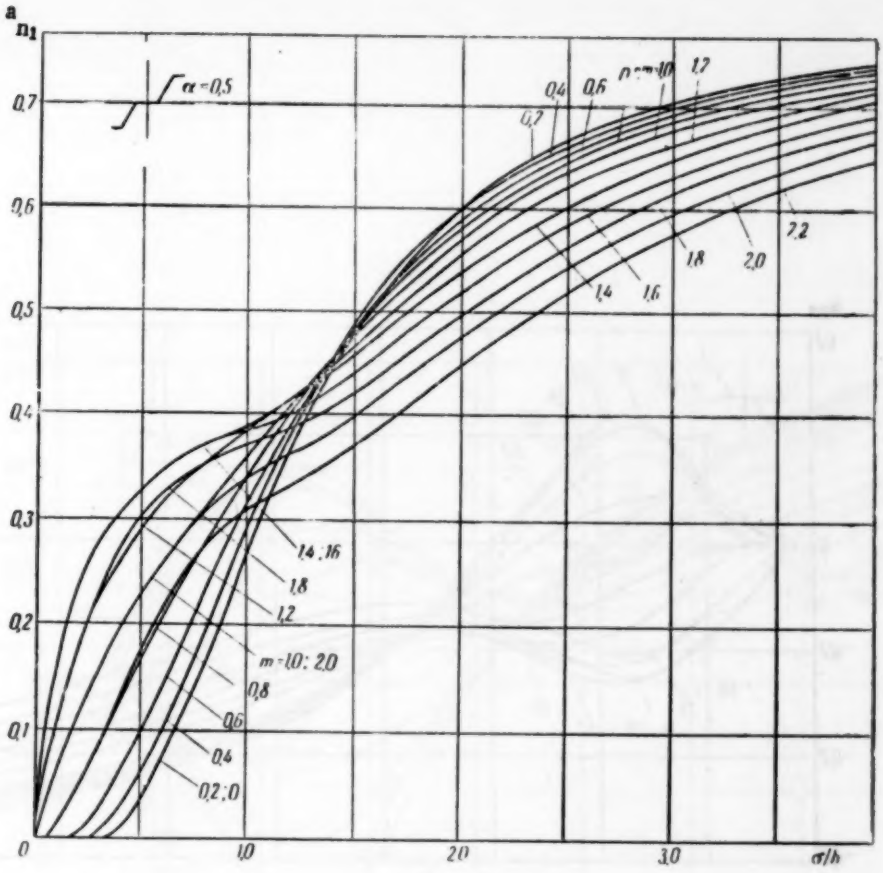

a
n1
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
α=0,5
0,4
0,6
0,7
1,7
1,8
2,0
2,2
1,4
1,6
1,4 ;1,6
1,8
1,2
m=1,0; 2,0
0,8
0,6
0,4
0,2; 0
1,0
2,0
3,0
σ/h

Fig. 16

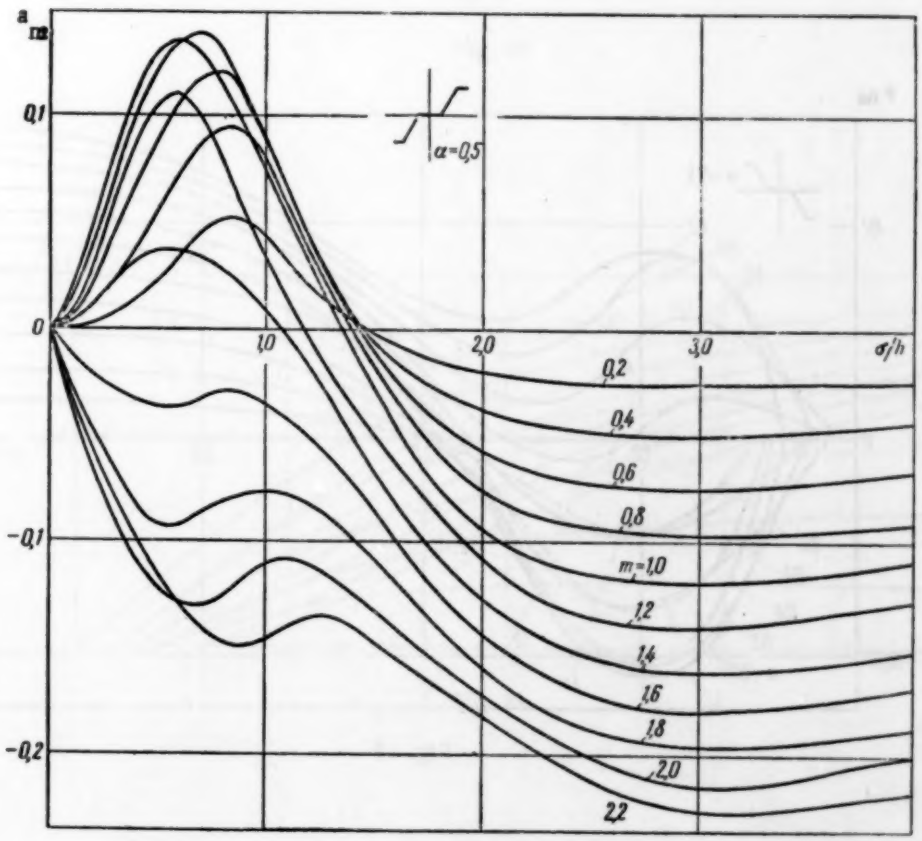

a
m
0,1
0
-0,1
-0,2
α=0,5
1,0
2,0
3,0
σ/h
0,2
0,4
0,6
0,8
m=1,0
1,2
1,4
1,6
1,8
2,0
2,2

Fig. 17

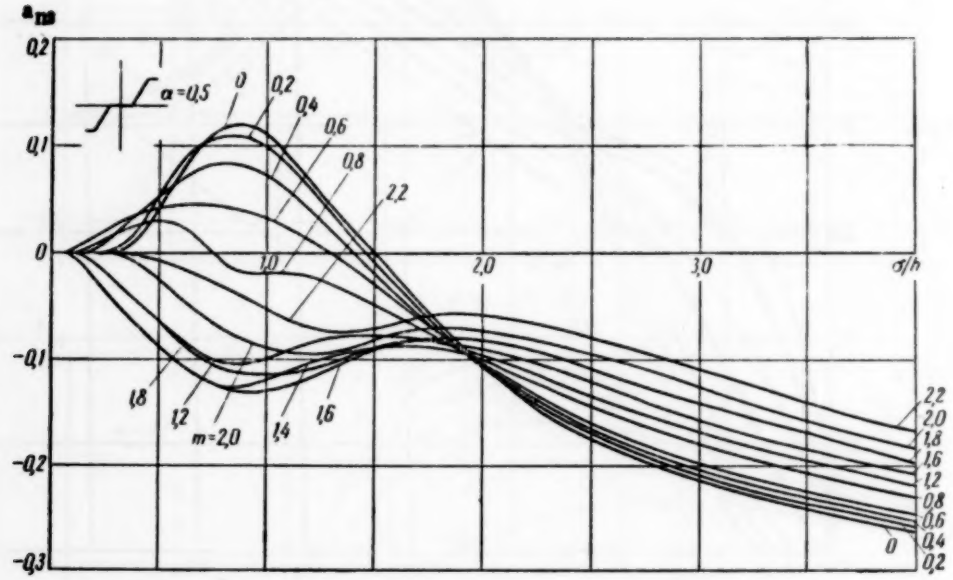

a_{m3}
0,2
0,1
0
-0,1
-0,2
-0,3
α=0,5
0
0,2
0,4
0,6
0,8
2,2
1,0
2,0
3,0
σ/h
1,8
1,2
m=2,0
1,4
1,6
2,2
2,0
1,8
1,6
1,2
0,8
0,6
0,4
0,2

Fig. 18

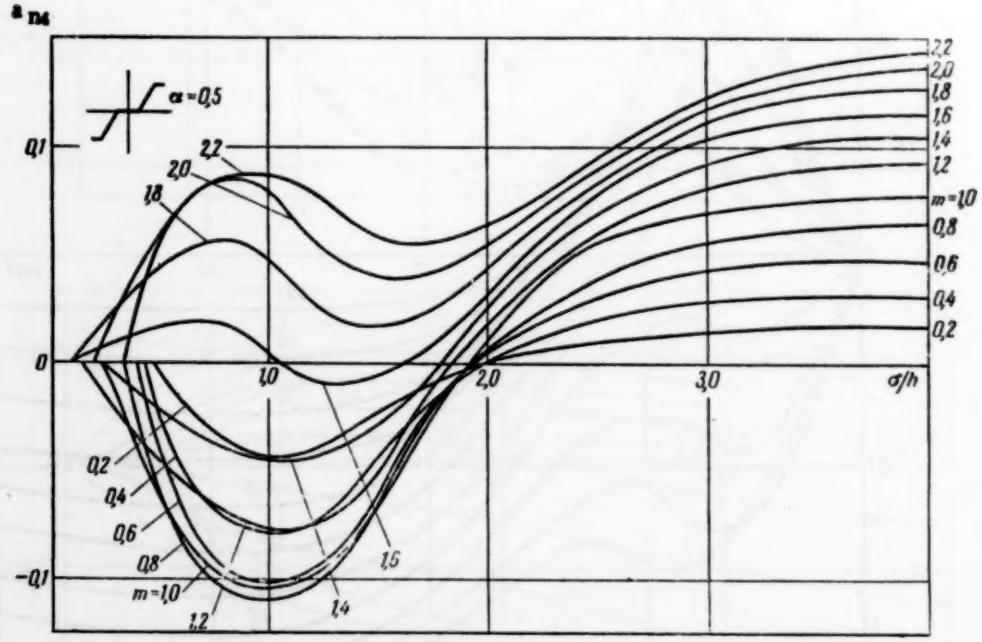

a_{m4}
α=0,5
0,1
0
-0,1
2,2
2,0
1,8
1,0
2,0
3,0
σ/h
0,2
0,4
0,6
0,8
m=1,0
1,2
1,4
1,6
2,2
2,0
1,8
1,6
1,4
1,2
m=1,0
0,8
0,6
0,4
0,2

Fig. 19

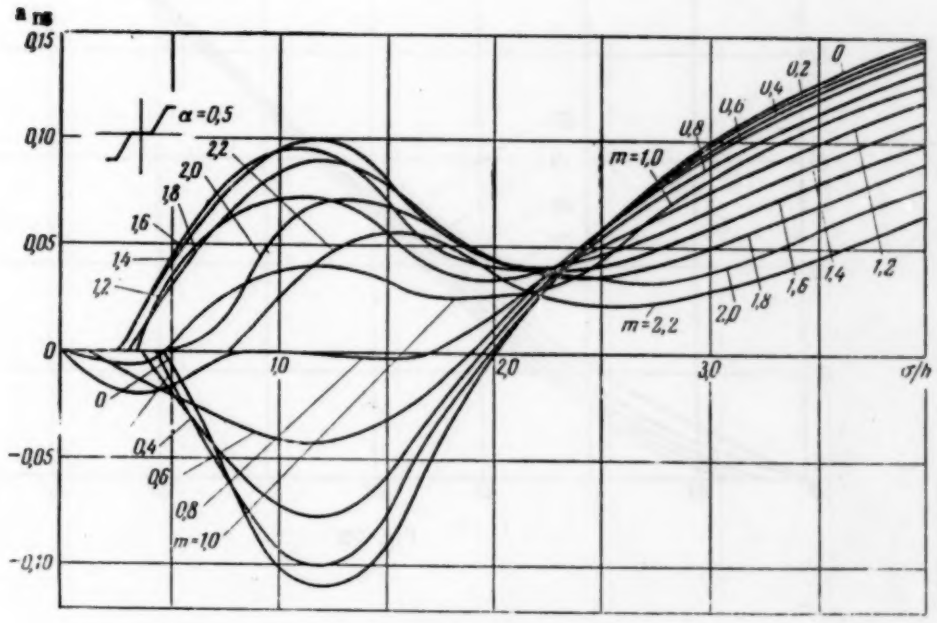

a rs
0,15
0,10
0,05
0
-0,05
-0,10
α=0,5
2,2
2,0
1,8
1,6
1,4
1,2
0
0,4
0,6
0,8
m=1,0
0,2
0,4
0,6
0,8
m=1,0
m=2,2
2,0
1,8
1,6
1,4
1,2
1,0
2,0
3,0
σ/h

Fig. 20

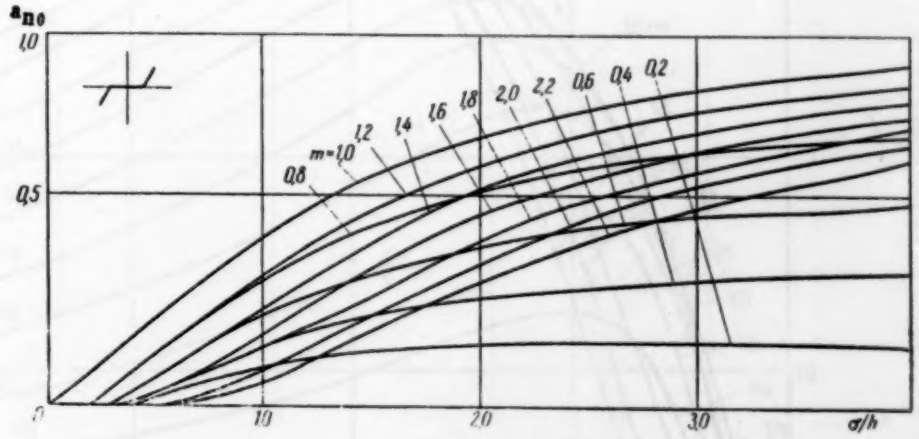

a no
1,0
0,5
0
m=1,0
0,8
1,2
1,4
1,6
1,8
2,0
2,2
0,6
0,4
0,2
1,0
2,0
3,0
σ/h

Fig. 21

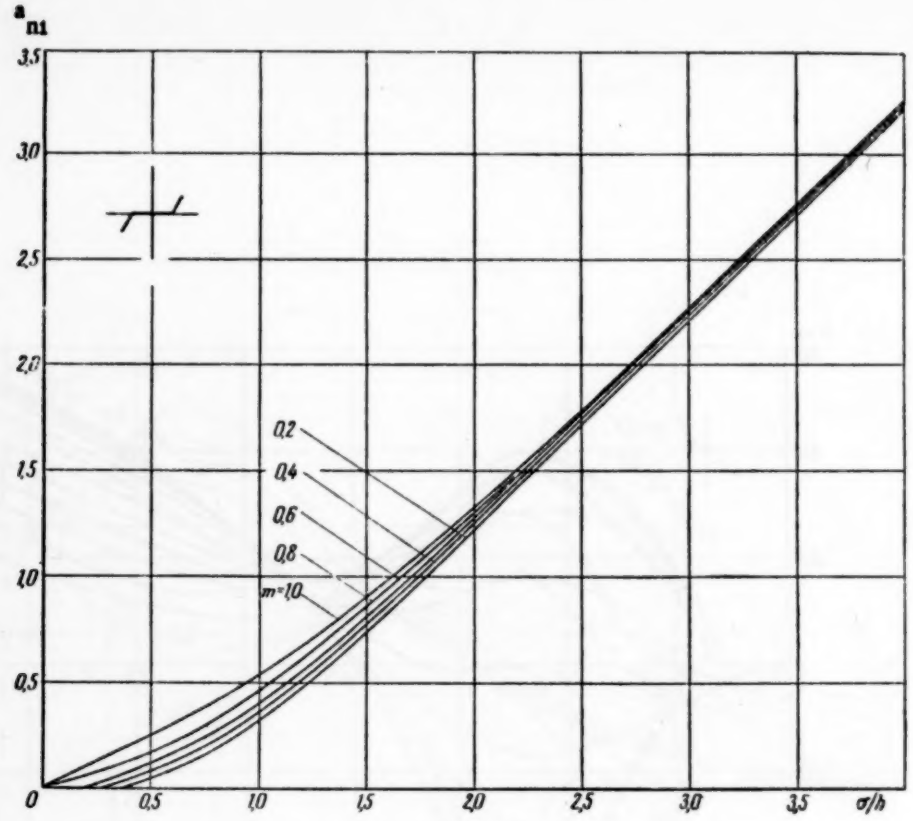

a n1
3,5
3,0
2,5
2,0
1,5
1,0
0,5
0
0,5
1,0
1,5
2,0
2,5
3,0
3,5
σ/h
0,2
0,4
0,6
0,8
m=1,0

Fig. 22

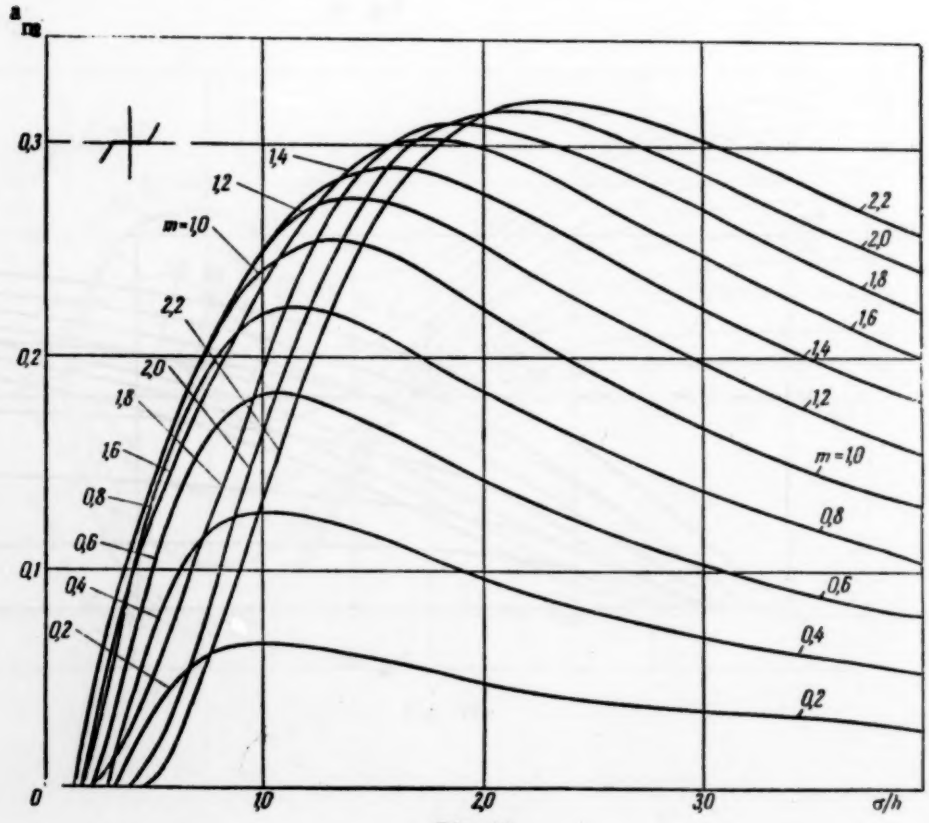

a n2
0,3
0,2
0,1
0
1,0
2,0
3,0
σ/h
1,4
1,2
m=1,0
2,2
2,0
1,8
1,6
0,8
0,6
0,4
0,2

Fig. 23

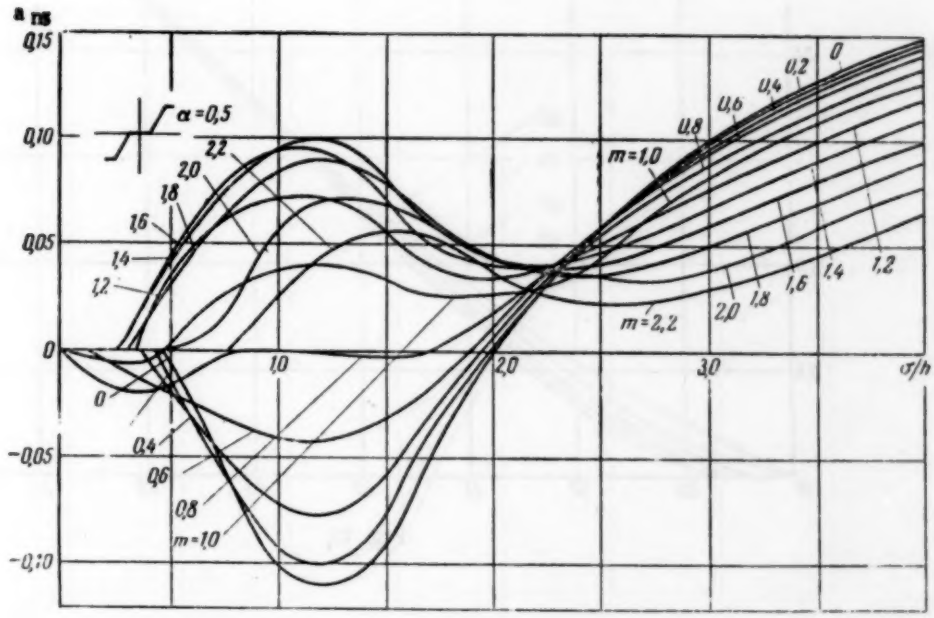

a ns
0,15
0,10
0,05
0
-0,05
-0,10
α=0,5
2,2
2,0
1,8
1,6
1,4
1,2
0
0,4
0,6
0,8
m=1,0
0,2
0,4
0,6
0,8
m=1,0
1,2
1,4
1,6
1,8
2,0
m=2,2
1,0
2,0
3,0
σ/h

Fig. 20

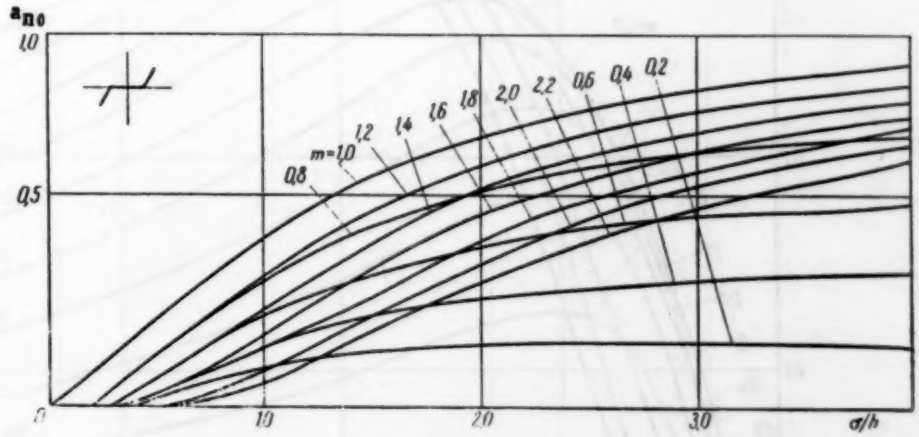
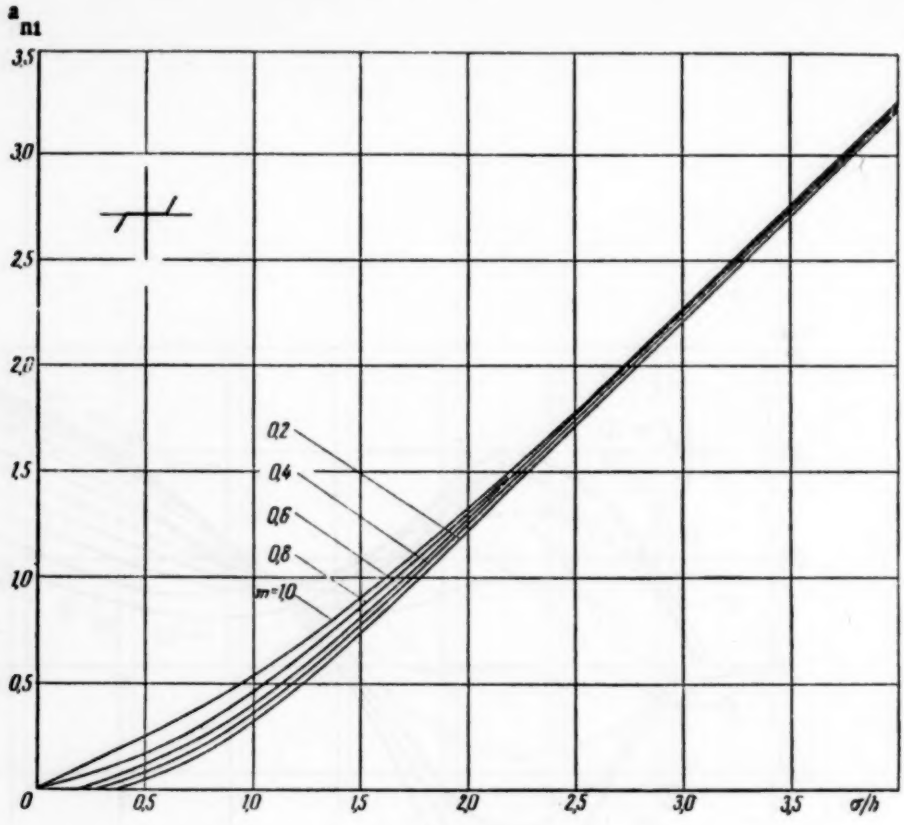

a n0
1,0
0,5
0
m=1,0
0,8
1,2
1,4
1,6
1,8
2,0
2,2
0,6
0,4
0,2
1,0
2,0
3,0
σ/h

Fig. 21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2,5
2,0
1,5
1,0
0,5
0
0,5
1,0
1,5
2,0
2,5
3,0
3,5
σ/h
0,2
0,4
0,6
0,8
m=1,0

Fig. 22

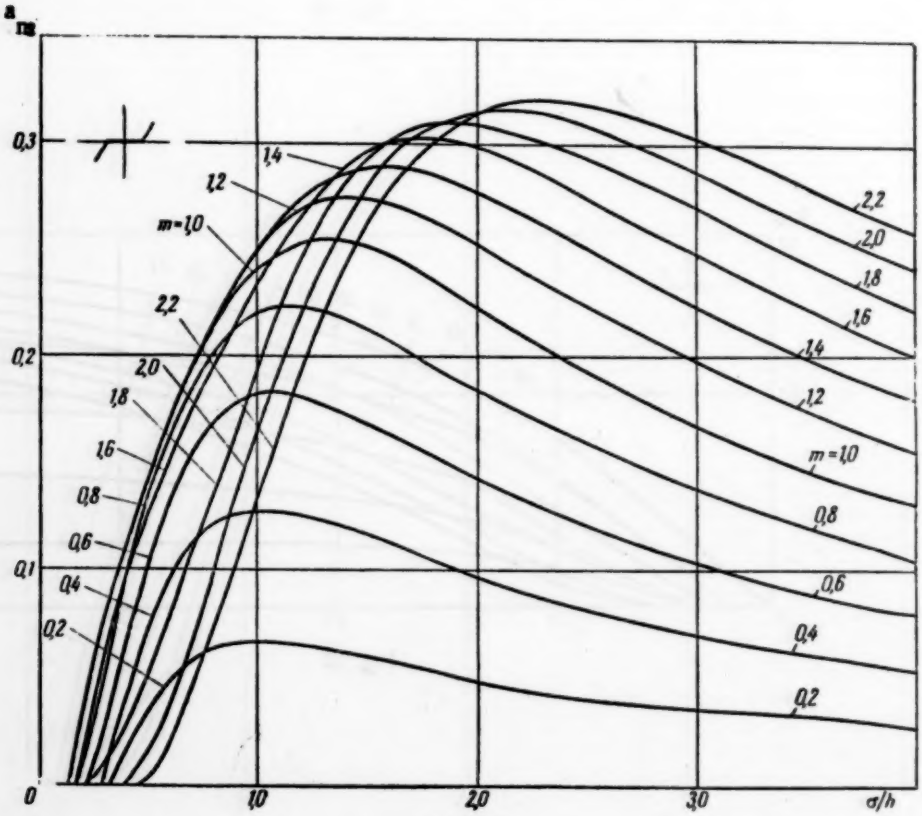
a_{n2}
0,3
0,2
0,1
0
1,0
2,0
3,0
σ/h
1,4
1,2
m=1,0
2,2
2,0
1,8
1,6
0,8
0,6
0,4
0,2

Fig. 23

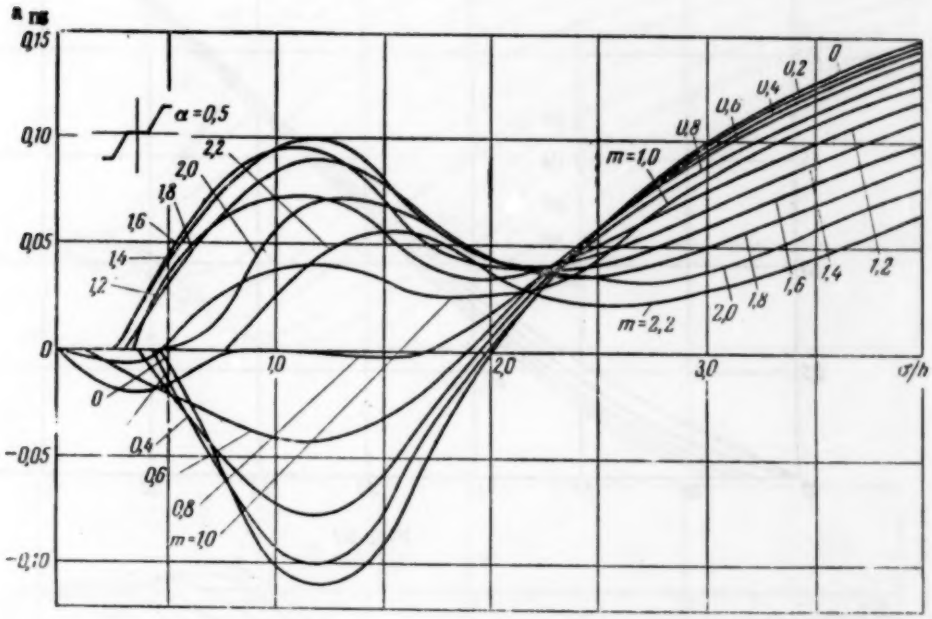
0,15
0,10
0,05
0
-0,05
-0,10
α=0,5
2,2
2,0
1,8
1,6
1,4
1,2
0
0,4
0,6
0,8
m=1,0
0,2
0,4
0,6
0,8
m=1,0
m=2,2
2,0
1,8
1,6
1,4
1,2
1,0
2,0
3,0
σ/h

Fig. 20

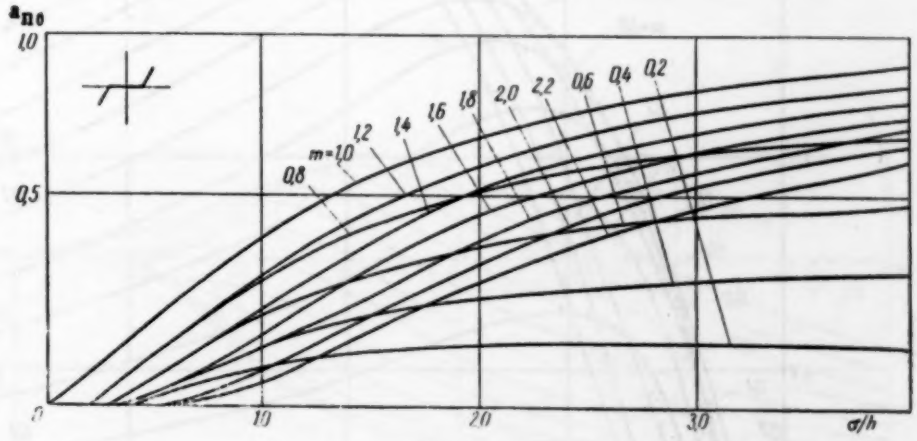
1,0
0,5
0
0,8
m=1,0
1,2
1,4
1,6
1,8
2,0
2,2
0,6
0,4
0,2
1,0
2,0
3,0
σ/h

Fig. 21

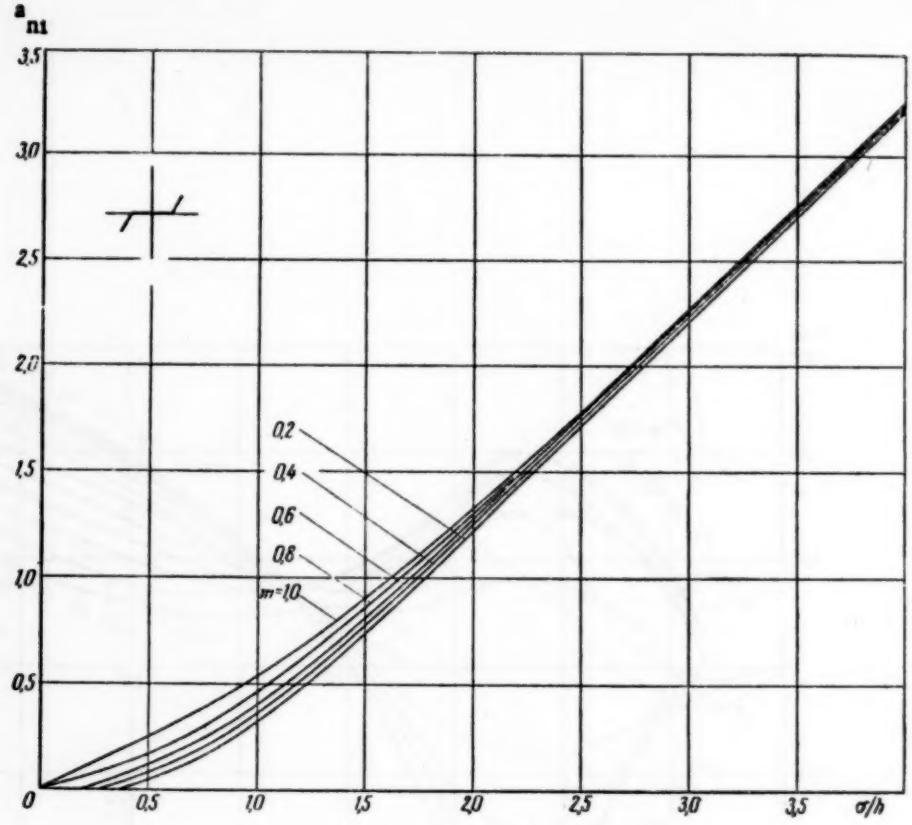
a n1
3,5
3,0
2,5
2,0
1,5
1,0
0,5
0
0,5
1,0
1,5
2,0
2,5
3,0
3,5
σ/h
0,2
0,4
0,6
0,8
m=1,0

Fig. 22

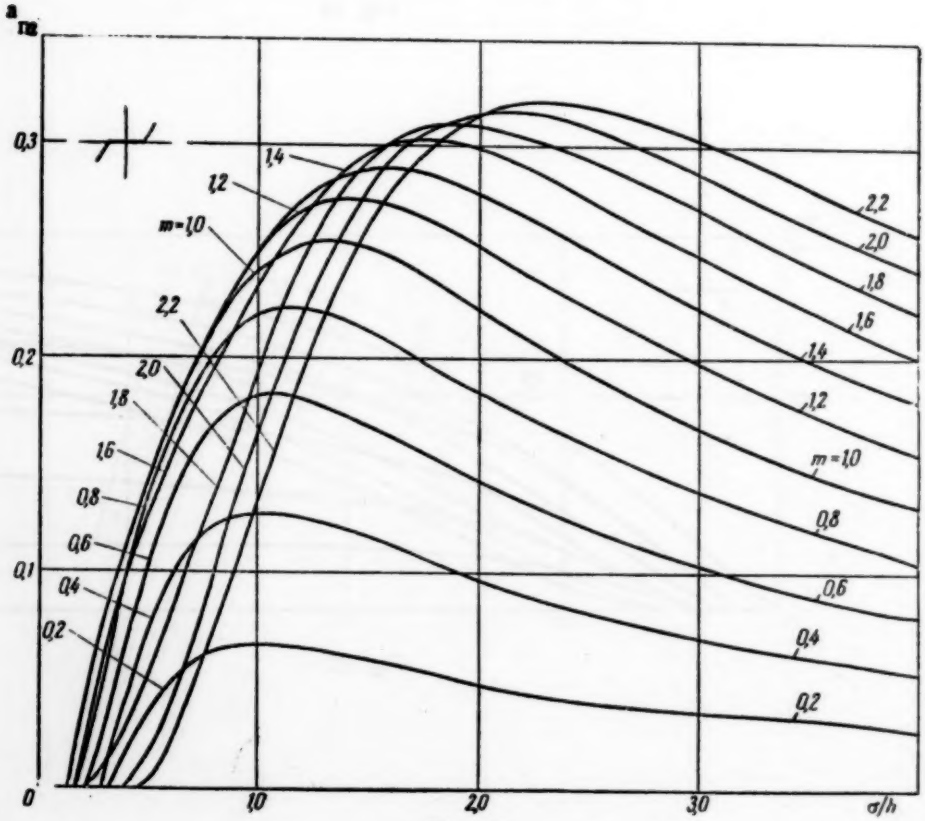
a n2
0,3
0,2
0,1
0
1,0
2,0
3,0
σ/h
1,4
1,2
m=1,0
2,2
2,0
1,8
1,6
0,8
0,6
0,4
0,2

Fig. 23

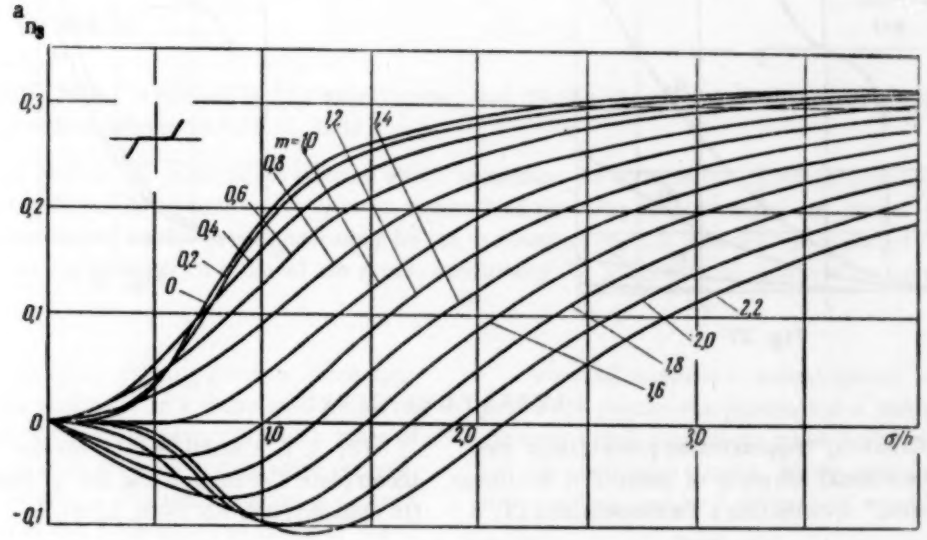

Fig. 24

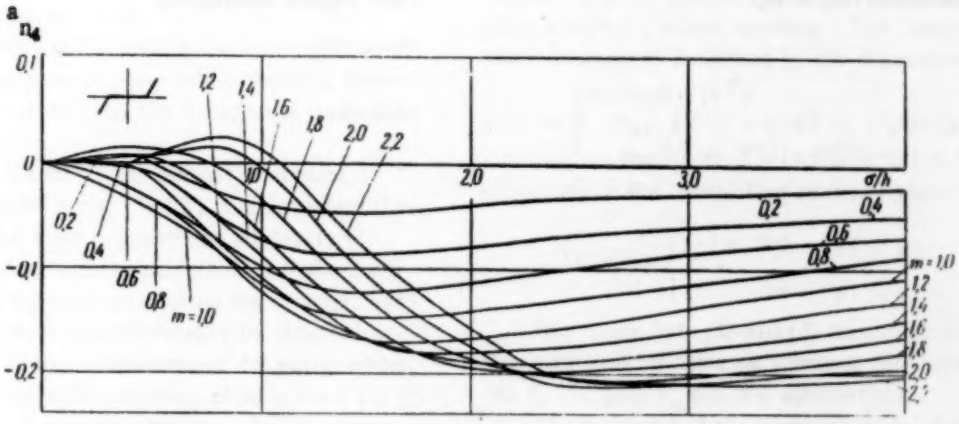

Fig. 25

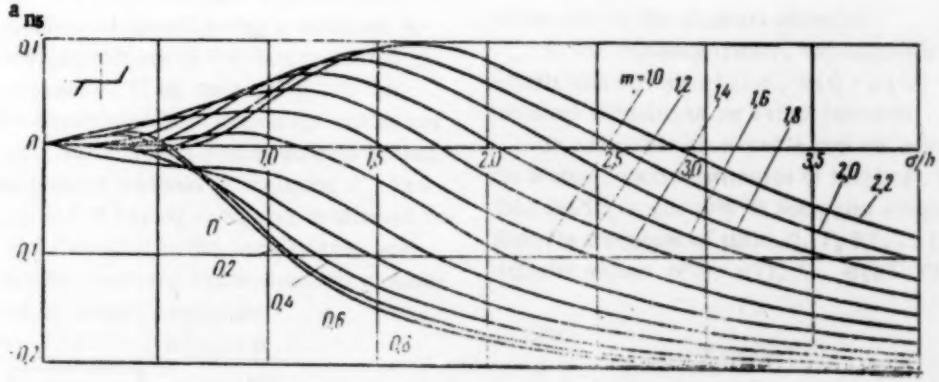

Fig. 26

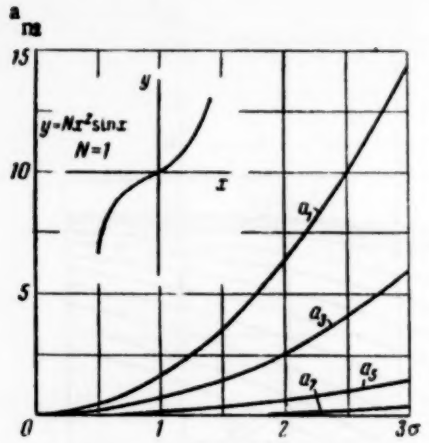

Fig. 27

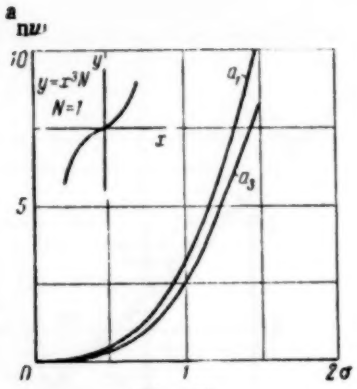

Fig. 28

LITERATURE CITED

[1] I. E. Kazakov, "Approximate probabilistic analysis of the operational accuracy of essentially nonlinear automatic systems," Avtomatika i Telemekhanika 17, 5 (1956). •

[2] H. Cramer, Mathematical Methods of Statistics [Russian translation] (IL, 1948).

[3] E. E. Slutskii, Tables for the Computation of the Incomplete Γ-Function and the χ^2 Probability Function [in Russian] (Izd. AN SSSR, 1950).

•See English translation.

PROCESSES OF FINITE DURATION IN PULSE-WIDTH SYSTEMS

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(Moscow)

(Translated from: Avtomatika i Telemekhanika, Vol. 21, No. 2, pp. 201-208, February, 1960)
Original article submitted July 18, 1959

Equations are derived for pulse-width systems which determine the form of the transient responses which terminate in a finite time. For second-order systems, methods of realizing such processes are given. It is suggested that the complicated nonlinearities necessary for the realization of such processes be replaced by linear elements. Results of experiments on a model are given, confirming the possibility of such replacements.

Today, much attention is being given to those optimal processes which terminate in a finite and, moreover, a minimal interval of time. In particular, such types of processes may be realized in relay and in pulse systems.

The duration of such a transient response is determined by the order of the differential equation of the system's continuous portion [1, 2]. Finite-duration processes in pulse-amplitude systems with amplitude limitations were considered in [3, 4].

The present work is devoted to the investigation of finite-duration processes in pulse-width systems, where the pulse height is given, and the duration is limited by the repetition interval.

In order that the transient response terminate at some point of discrete time, $t = l\,T$, it is necessary that, at this moment, the error quantity and its first (n-1) derivatives, inclusive, equal zero, where n is the order of the differential equation describing the system's continuous portion. These conditions may be satisfied by the proper choice of the n durations of the pulses which act on the system up to the moment of time $t = l\,T (l \geq n)$. Thus, with a limited external stimulus, the transient response can terminate after n cycles of the pulsed (sampled-data) element, just as in sampled-data systems with amplitude modulation. We shall call such a process an n-cycle one. The problem of investigating a transient response of finite duration reduces to the determination of the form of this process and of its realization. In this work we derive the equations which define the indicators of such a process and, for a system described by a second-order equation, we present methods of realizing it. Experiments on a type MN-7 analog computer confirmed the results obtained, and showed that the complicated nonlinearities necessary for realizing finite-duration processes may be replaced by simple amplifiers.

Conditions for Finite Duration

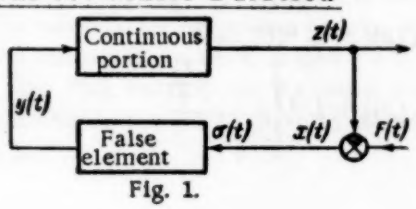

Fig. 1.

We shall consider a control system which is comprised of a continuous portion and a pulse (sampled-data) element which carried out pulse-width modulation (Fig. 1). Let the continuous portion, which contains an integrating link, be described by a differential equation of the form

$$a_n z^{(n)}(t) + a_{n-1} z^{(n-1)}(t) + \cdots + a_1 z'(t) = y(t), \quad (1)$$

where z(t) is the system's output quantity and y(t) is the pulse element's output quantity. The operation of this latter element is described by the expression

$$y(t) = \begin{cases} k_p \operatorname{sign} \sigma(kT) \\ \quad \text{for } kT \leq t < kT + T\Omega[\sigma(kT)], \\ 0 \quad \text{for } kT + T\Omega[\sigma(kT)] < t < kT + T, \end{cases} \quad (2)$$

where σ(t) is the controlling stimulus and

$$\Omega(\sigma) = \begin{cases} \frac{1}{\varkappa}|\sigma| & \text{for } |\sigma| < \varkappa, \\ 1 & \text{for } |\sigma| > \varkappa. \end{cases} \quad (3)$$

It is clear from (2) and (3) that the pulse element is characterized by three parameters: the repetition interval T, the gain k_p and the sensitivity $\varkappa$.

At the output of the comparator we shall have the quantity

$$x(t) = F(t) - z(t), \quad (4)$$

where F(t) is the external stimulus.

In the ordinary system, the controlling signal σ(t) equals this difference, i.e., $\sigma(t) = x(t)$. We assume the external stimulus to be a step function.

In order that an n-cycle process occur in the system for a step-function stimulus of height β, it is necessary that the input quantity to the pulse element σ(t) at the discrete moments of time, 0, T, 2T, .., (n-1)T assume definite values σ(0), σ(T), ..., σ(nT-T) such that, at

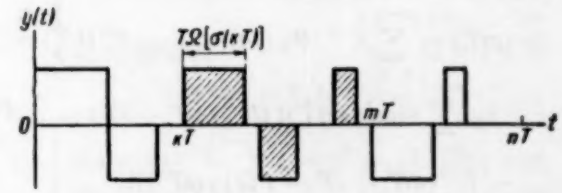

Fig. 2.

time t = nT, the system's output quantity z(t) take the value β and its derivatives, up to and including the (n-1)-th, vanish. Thus, the input quantity y(t) of the continuous portion is a sequence of n rectangular pulses (Fig. 2) whose durations are determined from the conditions for a finite duration of the process:

$$z(nT) = \beta, \quad z^{(j)}(nT) = 0 \quad (j = 1, 2, \ldots, n-1). \tag{5}$$

We now express the $z^{(j)}(mT)$ (j = 0,1,2,...,n-1) in terms $z^{(j)}(kT)$ (j = 0,1,2,..,n-1), where m and k are arbitrary numbers, with $n \geq m > k \geq 0$. It is obvious that the quantities $z^{(j)}(mT)$ can be found as the reaction of the continuous portion to the sequence of rectangular pulses shown hatched on Fig. 2, with initial conditions $z^{(j)}(kT)$ (j = 0,1,2,...,n-1).

If we apply the Lapace transform [5] to (1) with the initial conditions given above, and solve the expression thus obtained with respect to the transform of the output quantity, i.e., with respect to Z(p), we find that

$$Z(p) = \sum_{i=0}^{n-1} A_i(p)\, z^{(i)}(kT) + K(p)\, Y_{km}(p),$$

where

$$A_i(p) = \frac{a_{i+1} + a_{i+2}p + \cdots + a_n p^{n-i-1}}{a_1 p + \cdots + a_n p^n},$$

$$K(p) = \frac{1}{a_1 p + \cdots + a_n p^n},$$

and $Y_{km}(p)$ is the transform of the hatched portion of the sequence of rectangular pulses (Fig. 2),

$$Y_{km}(p) = k_p \sum_{i=k}^{m-1} \operatorname{sign}[\sigma(iT)]\, e^{-pT(i-k)}\, \frac{1 - e^{-pT\Omega[\sigma(iT)]}}{p}.$$

Using the well-known rules [5] for finding the transforms of the derivatives, we get

$$L\{z^{(j)}(t)\} = \sum_{i=0}^{n-1} B_{ji}(p)\, z^{(i)}(kT) + \tag{6}$$

$$+ k_p p^j K(p) \sum_{i=k}^{m-1} \operatorname{sign}[\sigma(iT)] e^{-pT(i-k)}\, \frac{1 - p^{-pT\Omega[\sigma(iT)]}}{p},$$

where

$$B_{ji}(p) = \begin{cases} p^j\, \dfrac{a_{i+1} + a_{i+2}p + \cdots + a_n p^{n-i-1}}{a_1 p + \cdots + a_n p^n} & \text{for } j \leqslant i, \\ -p^{j-i-1}\, \dfrac{a_1 p + \cdots + a_i p^i}{a_1 p + \cdots + a_n p^n} & \text{for } j > i. \end{cases} \tag{7}$$

If we go from (6) to the time-domain function for t = =(m-k)T, we obtain the relationship connecting the quantity z(t) and its derivatives at time t = mT with the quantity z(t) and its derivatives at time t = kT.

$$z^{(j)}(mT) = \sum_{i=0}^{n-1} L^{-1}\{B_{ji}(p)\}_{t=(m-k)T} z^{(i)}(kT) + \tag{8}$$

$$+ k_p \sum_{i=k}^{m-1} \operatorname{sign}[\sigma(iT)]\, \{h^{(j)}(mT - iT) - h^{(j)}(mT - iT - T\Omega[\sigma(iT)])\},$$

where $h^{(j)}(t)$ is the jth derivative of the transient-response function of the system's continuous portion.

This last expression is necessary for us in our investigation of second-order systems. Here, we shall make use of it in order to express analytically the conditions for finite duration of transient response (5) for a system of any order. By setting m = n and k = 0 in (8), and by taking into account that $z^{(j)}(0) = 0$ (j = 0,1,2,..,n-1), we find that

$$\begin{gathered} k_p \sum_{i=0}^{n-1} \operatorname{sign}\sigma(iT)\, \{h(nT - iT) - \\ - h(nT - iT - T\Omega[\sigma(iT)])\} = \beta, \\ \sum_{i=0}^{n-1} \operatorname{sign}\sigma(iT)\, \{h^{(j)}(nT - iT) - \\ - h^{(j)}(nT - iT - T\Omega[\sigma(iT)])\} = 0 \\ (j = 1, 2, \ldots, n-1). \end{gathered} \tag{9}$$

In order for an n-cycle process to be realized, the controlling stimuli σ(t) must assume, at times 0,T,2T,..., (n-1)T, the values defined by (9). These equations are transcendental so that, just as in the case of relay systems, it is impossible, in the general case, to solve them for the σ(iT) (i = 0,1,2,..,n-1) or to state a general method for realizing such a process. We therefore limit ourselves in the sequel to the consideration of special cases, specifically, to second-order systems.

Second-Order Systems

Let the continuous portion be described by the differential equation

$$T_1 z''(t) + z'(t) = k y(t).$$

By comparing this expression with (1), we note that

$$n = 2, \quad a_2 = \frac{T_1}{k}, \quad a_1 = \frac{1}{k}, \quad K(p) = \frac{k}{p(T_1 p + 1)},$$

from whence

$$h(t) = L^{-1}\left\{\frac{1}{p} K(p)\right\} = L^{-1}\left\{\frac{k}{p^2(T_1 p + 1)}\right\} = \tag{10}$$

$$= k\left[t - T_1\left(1 - e^{-\frac{t}{T_1}}\right)\right],$$

$$h'(t) = k\left(1 - e^{-\frac{t}{T_1}}\right). \tag{11}$$

If we take the inverse Laplace transform [5] of (7), we find that

$$L^{-1}\{B_{00}(p)\} = L^{-1}\left\{\frac{\frac{1}{k} + \frac{T_1}{k} p}{\frac{1}{k} p + \frac{T_1}{k} p^2}\right\} = 1[t], \tag{12}$$

$$L^{-1}\{B_{01}(p)\} = L^{-1}\left\{\frac{\frac{T_1}{k}}{\frac{1}{k} p + \frac{T_1}{k} p^2}\right\} = T_1\left(1 - e^{-\frac{t}{T_1}}\right), \tag{13}$$

$$L^{-1}\{B_{10}(p)\} = L^{-1}\{0\} = 0, \tag{14}$$

$$L^{-1}\{B_{11}(p)\} = L^{-1}\left\{p\, \frac{\frac{T_1}{k}}{\frac{1}{k} p + \frac{T_1}{k} p^2}\right\} = e^{-\frac{t}{T_1}}. \tag{15}$$

If we substitute (10) and (11) in (9) for n = 2, and carry out some elementary transformations, we can write the conditions for finite duration in the following form:

$$\operatorname{sign}\sigma(T)\left(e^{\frac{T}{T_1}\Omega[\sigma(T)]}-1\right)+ \\ +\operatorname{sign}\sigma(0)\left(e^{\frac{T}{T_1}\Omega[\sigma(0)]}-1\right)e^{-\frac{T}{T_1}}=0 \tag{16}$$

and, by virtue of (16),

$$x(0)=\beta=kk_pT\operatorname{sign}\sigma(T)\,\Omega[\sigma(T)]+ \\ +kk_pT\operatorname{sign}\sigma(0)\,\Omega[\sigma(0)]. \tag{17}$$

In accordance with (3), the parenthesized terms in (16) will be greater than zero and, in order that the left member of (16) equal zero, it is necessary that the quantities $\sigma(T)$ and $\sigma(0)$ have opposite signs. With this taken into account, we solve (16) for $\Omega[\sigma(T)]$ and substitute the expression thus obtained in (17). The result is

$$x(0)=N_0[\sigma(0)]=kk_pT\operatorname{sign}\sigma(0)\left\{\Omega[\sigma(0)]- \right. \\ \left.-\frac{T_1}{T}\ln\left(e^{-\frac{T}{T_1}+\frac{T}{T_1}\Omega[\sigma(T)]}-e^{-\frac{T}{T_1}}+1\right)\right\}, \tag{18}$$

Where the function $\Omega[\sigma(0)]$ is defined by (3). The inverse relationship, i.e.,

$$\sigma(0)=N_0^{-1}[x(0)], \tag{19}$$

is the relationship sought between the error x(t) and the input function $\sigma(t)$ of the pulse element at time t = 0.

At time t = T, the relationship between x(T) and $\sigma(T)$ has a different form. To find it, we use (8) for k=1 and m = n = 2, in which we substitute (10)-(15)for t = -(m-k)T = T. If we substitute the values thus obtained for z(2T) and z'(2T), expressed in terms of z(T) and z'(T), in conditions (5) for finite duration, and solve the equations obtained with respect to z(T) and z'(T), we get

$$z(T)=\beta-kk_pT\operatorname{sign}\sigma(T)\times \\ \times\left\{\Omega[\sigma(T)]-\frac{T_1}{T}\left(e^{\frac{T}{T_1}\Omega[\sigma(T)]}-1\right)\right\}=\beta-N_1[\sigma(T)], \tag{20}$$

$$z'(T)=-kk_p\operatorname{sign}\sigma(T)\left(e^{\frac{T}{T_1}\Omega[\sigma(T)]}-1\right)= \\ =-N_2[\sigma(T)]. \tag{21}$$

By noting that, for β = const, x(T) = β -z(T) and x'(T) = -z'(T), we find from these expressions that $x(T)=N_1[\sigma(T)]$, $x'(T)=N_2[\sigma(T)]$, or

$$\sigma(T)=N_1^{-1}[x(T)], \tag{22}$$

$$\sigma(T)=N_2^{-1}[x'(T)]. \tag{23}$$

Relationships (22) and (23) define the relationship between the values of the controlling function and the error, or the velocity of the error, at time t = T. It follows from (19), (22), and (23) that the output quantity of the controlling device can be given by the following two methods:

$$\sigma(t)=\begin{cases}N_0^{-1}[x(0)] & \text{for } t=0,\\ N_1^{-1}[x(T)] & \text{for } t=T\end{cases} \tag{24}$$

and

$$\sigma(t)=\begin{cases}N_0^{-1}[x(0)] & \text{for } t=0,\\ N_2^{-1}[x'(T)]=-N_2^{-1}[z'(T)] & \text{for } t=T.\end{cases} \tag{25}$$

In the intervals between the cycle times (the discrete time moments), $\sigma(t)$ can assume arbitrary values.

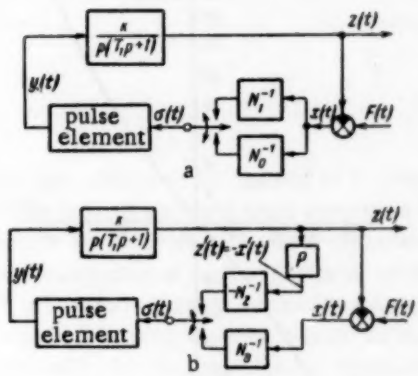

Fig. 3.

In accordance with (24) or (25), the controlling device is comprised of two nonlinear elements and a relay, which switches the error signal from one nonlinear element to the other (Fig. 3, and b). Realization (24) (Fig. 3, a) is worse than realization (25) (Fig. 3, b) since, if the external stimulus varies, not by jumps but arbitrarily, then at time t = T the comparator's output quantity in the first case will not correspond to a two-cycle process, and the system will not process in two cycles that stimulus which was measured for t = 0. In the second case (25) (Fig. 3,b), at cycle time t = T, control is effected by the velocity of the system's output quantity, and on this the variations of the external stimulus have no influence.

Example. As a numerical example, we consider a system in which the continuous portion is described by the differential equation

$$Z''+Z'=y(t),$$

i.e., T_1 = 1 sec, k = 1 and the pulse element has the following parameters: k_p = 20v, κ = 20v and T = 1 sec.

To this system, we apply the second variant of the realization of a finite-duration process. By substituting the parameter values given above for the continuous portion and the pulse element in Expressions (18) and (21), and by taking (3) into account, we find that

$$x(0)=N_0[\sigma(0)]= \\ =\begin{cases}\sigma(0)-20\operatorname{sign}\sigma(0)\ln\left(0{,}368\,e^{\frac{|\sigma(0)|}{20}}+0{,}632\right) & \text{for } |\sigma(0)|<20,\\ 10{,}3\operatorname{sign}\sigma(0) & \text{for } |\sigma(0)|>20,\end{cases}$$

$$-Z'(T)=N_2[\sigma(T)]= \\ =\begin{cases}20\operatorname{sign}\sigma(T)\left(e^{\frac{|\sigma(T)|}{20}}-1\right) & \text{for } |\sigma(T)|<20,\\ 34{,}4\operatorname{sign}\sigma(T) & \text{for } |\sigma(T)|>20.\end{cases}$$

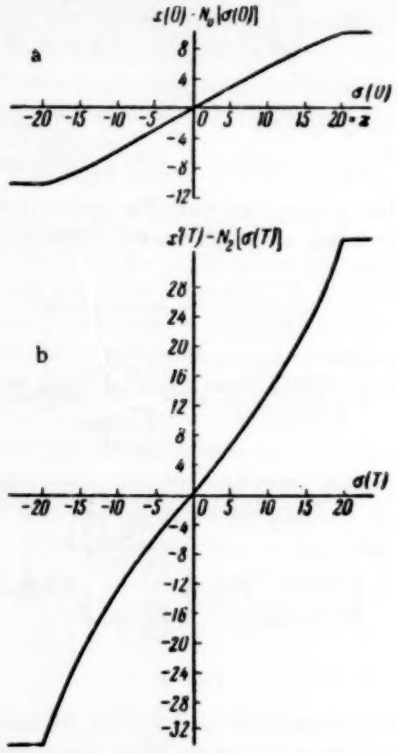

Fig. 4.

The graphs of the relationships $x(0) = N_0[\sigma(0)]$ and $-Z'(T) = N_2[\sigma(T)]$ are given on Fig. 4, a and b. For the realization of a two-cycle process with nonlinearities inverse to these, it is necessary to use control by the scheme shown in Fig, 3,b in the system.

It turned out, with empirical investigation on a model, that these nonlinearities can be replaced by amplifiers which roughly approximate them. It is clear with this that the saturation zone can be ignored, since the pulse element itself, according to (3), possesses saturation for values of the input signal $\sigma(t)$ which correspond to the onset of saturation in the nonlinearities N_0 and N_2. Thus, in the given example, the nonlinearities $N_0^{-1}[x(t)]$ and $N_2^{-1}[-Z'(t)]$, shown in Fig. 3,b can be replaced by amplifiers with gains approximately equal to 2.5 and 0.6 respectively.

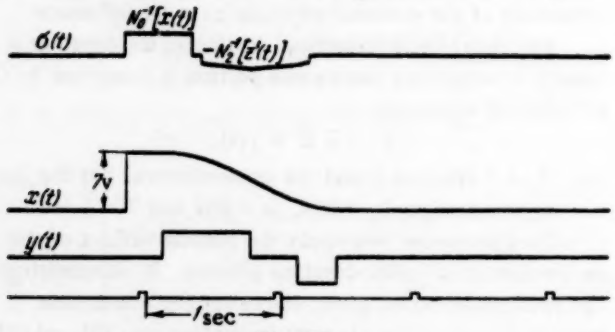

Fig. 5a.

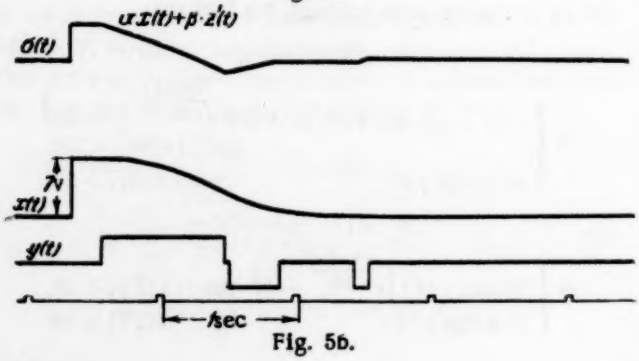

Fig. 5b.

Figure 5,a shows the oscillogram of the transient response of a finite-duration process in the system for a stimulus $F(t) = \beta \cdot 1[t] = 7v$, from which it is clear that the transient response terminates after two cycles of the pulse element, while the ordinary process continues for approximately six cycles. By using the standard linear-correction methods, for example, the introduction of derivative control, one can significantly improve the transient response, but this improved response will last longer than two cycles (Fig. 5,b).

In conclusion, we make several remarks about the behavior of such systems for large amplitudes of the external stimuli. From the oscillograms shown in Fig. 6,a and b, it is clear that, for $\beta = 15v$, the duration of the finite-duration transient response (Fig. 6,a) is comparable with the duration of the response in the system for which the relationship between stimulus magnitude and error velocity was selected in the very best way (Fig. 6,b). Thus, the method presented above for the realization of a two-cycle process becomes of little effect when the stimuli are large.

Equation (18) gives the relationship between the quantity which the system can process within two cycles, and the relative duration of the first corresponding pulse $\Omega[\sigma(0)]$, which cannot be greater than unity. Thus, the maximum amplitude of the external stimulus which the system can process in two cycles is determined by (18) for $\Omega[\sigma(0)]=1$, i.e.

$$x_{0\max} = kk_p T\left[1 - \frac{T_1}{T}\ln\left(2 - e^{-\frac{T}{T_1}}\right)\right]. \qquad (26)$$

In our numerical example, $x_{0\max} = 10.3$. For $F(t) = \beta > x_{0\max}$, the system processes $x_{0\max}$ during the first two cycles and, during the next two cycles, the quantity $\beta - x_{0\max}$. If this latter quantity is less than $x_{0\max}$, the process terminates after four cycles, otherwise it continues for even longer. Figure 7,a shows the approximate form of such a transient response. As is clear from this figure, the response consists of a series of two-cycle processes. In this case, even the introduction of a simple combination of the quantity and its derivatives can result it an improved transient response. This, however, occurs only for $\beta > x_{0\max}$.

It follows from (26) that the magnitude of $x_{0\max}$ is proportional to the repetition period T. If $\beta > x_{0\max}$, then, to obtain an even shorter finite-duration process, it is necessary that the first pulse not terminate at time $t = T$, but continue even longer, i.e., the system must operate as if the repetition period were 2T, not T. The approximate form of such a process is shown on Fig. 7,b. The dashed lines on Fig. 7,a and b show the variations of the

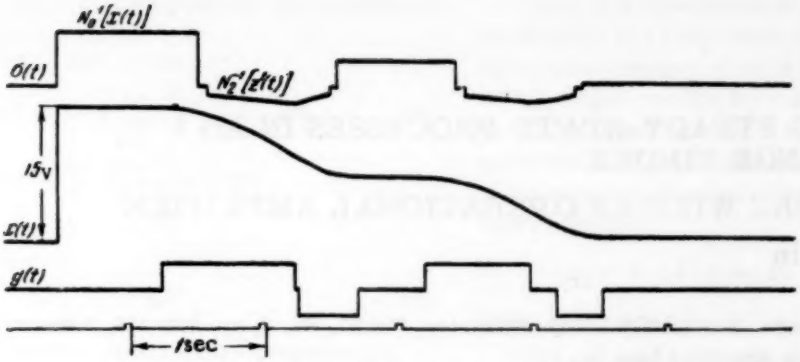

Fig. 6a.

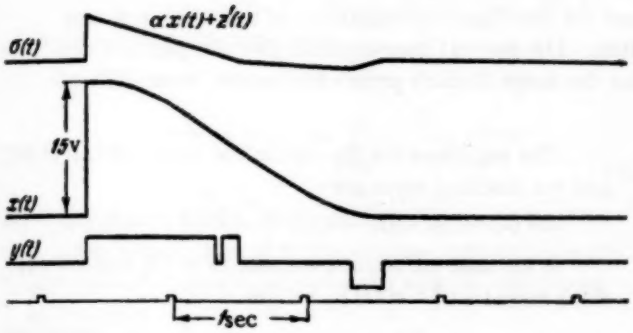

Fig. 6b.

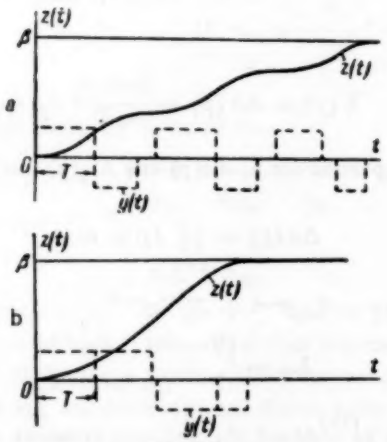

Fig. 7.

quantity $y(t)$. The total duration of the first and second, and also the third and fourth, pulses can be determined from the same expressions as in the case when $\beta < x_{0\max}$ if, in them, one substitutes 2T instead of T. For even greater values of β, the system must operate as if the repetition period were 3T, 4T, etc., as β increases.

For the realization of such still shorter transient responses, it is necessary that the controlling device incorporate several controlling devices, similar to those shown on Fig. 3, a and b, but designed for the repetition periods T, 2T, 3T, 4T, etc. as a function of the maximum possible value of the external stimulus. The switching of one or another of the controlling devices into the system must be implemented as a function of the magnitude of the initial error $x(0)$, which equals the external stimulating step β, and may be implemented by means of some supplementary device.

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TRANSIENT AND STEADY-STATE PROCESSES IN AN AUTOMATIC RANGE FINDER

II. RANGE FINDER WITH AN OPERATIONAL AMPLIFIER

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The dynamic properties of a range finder with an operational amplifier, including an integrating and an inertial link, are considered. In the general case, the investigation of the processes in a range finder with an operational amplifier, if with this one takes into account the step-function variations of the time-separator parameters, leads to cumbersom mathematical computations. The general investigation given in part I of this work [1] and the equations therein obtained for estimating the range finder's properties permit these difficulties to be avoided.

1. Properties of the Range Finder with an Operational Amplifier When It Is Investigated as a Continuous Automatic-Control System

An estimate of the effect of noise on a range finder of the simplest construction, not containing any operational amplifier, was given in [2]. In the present work, we start from the general-range-finder theory presented in [1]. Application of this theory to practically important concrete designs permits their dynamic properties to be estimated comparatively simply. We shall not treat the passage of random signals through the range finder in this paper.

In [1] we found the conditions under which the investigation of the processes in the range finder can be reduced to the investigation of processes in the continuous system described by (I.38)*. We shall assume here that these conditions are met†. We take a definite structure for the operational amplifier, for example, one for which (I.38) take the following form.

The equations for the time separator and the operational amplifier are

$$\begin{aligned}
&(D + a_{11})\,U_1(t) = \frac{2K_0 F_s}{\tau_{10} T}\,\Delta\alpha(t),\\
&a_{21}U_1(t) + (D + a_{22})\,U_2(t) = 0,\\
&a_{31}U_1(t) + a_{32}U_2(t) + (D + a_{33})\,U_3(t) = 0,\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
&\sum_{m=1}^{r-1} a_{rm}U_m(t) + (D + a_{rr})\,U_r(t) = 0.
\end{aligned} \tag{1}$$

The parameter a_{11} appertains to the time separator, and is expressed in terms of the time constants τ_{10} and τ_{11} by

$$a_{11} = \frac{\frac{\alpha}{\tau_{10}} + \frac{T-\alpha}{\tau_{11}}}{T}. \tag{2}$$

The equations for the controlled delay circuit (CDC) and the tracking error are

$$\Delta\vartheta(t) = \sigma U_r(t),\quad \Delta\alpha(t) + \Delta\vartheta(t) = \Delta\vartheta_1(t). \tag{3}$$

If we take the Lapace transforms of (1) and (3) and solve them for $L\{\Delta\alpha(t)\}$, we get

$$L\{\Delta\alpha(t)\} = \frac{\Delta a(p)}{X(p)}\,L\{\Delta\vartheta_1(t)\} - \sigma\sum_{k=1}^{r}\frac{A_{rk}(p)\,U_k(0)}{X(p)}, \tag{4}$$

where

$$X(p) = \Delta a(p) + \frac{2\sigma K_0 F_s}{\tau_{10} T}\,A_{r1}(p). \tag{5}$$

In expanded form, $\Delta a(p)$ and $A_{rk}(p)$ are given by

$$\begin{aligned}
&\Delta a(p) = \prod_{k=1}(p + a_{kk}),\\
&A_{rk}(p) = \delta_{rk}p^{r-1} + Z_1^{(rk)}p^{r-2} + \cdots + Z_{r-1}^{(rk)},\\
&\delta_{rk} = \begin{cases} 0 & \text{for } r \neq k,\\ 1 & \text{for } r = k,\end{cases}
\end{aligned} \tag{6}$$

where the $Z_j^{(rk)}$ ($j = 1,2,\ldots,r-1$) are constant real coefficients.

The form of the operator function $X(p)$, defined by (5), depends on the structure of the element A_{r1}. In its expanded form, the expression for $A_{r1}(p)$ is given as

$$A_{r1}(p) = (-1)^{r+1}\begin{vmatrix} a_{21}, & p + a_{22}, & 0, \ldots, 0\\ a_{31}, & a_{32}, & p + a_{33}, \ldots, 0\\ \cdots & \cdots & \cdots\\ a_{r1}, & a_{r2}, & a_{r3}, \ldots, a_{r,r-1}\end{vmatrix} = Z_1p^{r-2} + Z_2p^{r-3} + \cdots + Z_{r-1}, \tag{7}$$

*The Roman numeral I in parentheses means that the formula numeration refers to part I of the given work.

†Cf. [1], section 9 and (I.44).

where the Z_j ($j = 1,2,\ldots,r-1$) are constant real coefficients.

We introduce the notation

$$\Delta\alpha = \Delta\alpha_1 + \Delta\alpha_2,$$

$$L\{\Delta\alpha_1(t)\} = -\sigma \sum_{k=1}^{r} \frac{A_{rk}(p)}{X(p)} U_k(0), \qquad (8)$$

$$L\{\Delta\alpha_2(t)\} = \frac{\Delta a(p)}{X(p)} L\{\Delta\vartheta_1(t)\}.$$

We assume that

$$\Delta\vartheta_1(t) = \eta_s t^s, \qquad (9)$$

where the η_s are constant coefficients. Consequently,

$$L\{\Delta\alpha_2(t)\} = s!\,\frac{\eta_s \Delta a(p)}{p^{s+1} X(p)}. \qquad (10)$$

If the range finder is a stable system, then, for the steady-state mode, we obtain

$$\Delta\alpha_1 = 0,$$

$$\Delta\alpha_2(t) = s!\ \eta_s \sum_{k=0}^{s} \frac{G_k}{(s-k)!} t^{s-k}, \qquad (11)$$

$$G_k = \frac{1}{k!}\left[\frac{d^k}{dp^k}\frac{\Delta a(p)}{X(p)}\right]_{p=0}.$$

For large values of t, the greatest influence on the accuracy of range-finder operation will be manifested by the term

$$\Delta\alpha_{20}(t) = \eta_s G_0 t^s, \quad G_0 = \frac{\prod_{k=1}^{r} a_{kk}}{\prod_{k=1}^{r} a_{kk} + \frac{2\sigma K_0 E_s}{\tau_{10} T} A_{r1}(0)}. \qquad (12)$$

On the basis of (7), we have

$$A_{r1}(0) = (-1)^{r+1} \begin{vmatrix} a_{21}, & a_{22}, & 0, & \ldots, & 0 \\ a_{31}, & a_{32}, & a_{33}, & \ldots, & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ a_{r1}, & a_{r2}, & a_{r3}, & \ldots, & a_{r,r-1} \end{vmatrix} \qquad (13)$$

It follows from (12) and (13) that the steady-state accuracy of the range finder will be increased if the coefficients a_{kk} are decreased and the quantity $A_{r1}(0)$ is increased. According to (13), the increase of $A_{r1}(0)$ can be achieved by means of the operational amplifier parameters a_{jk} for which $j \neq k$.

To see more graphically how the coefficients a_{kk} ($k = 1,2,..,r$) affect tracking accuracy, we consider the case when the following equalities hold

$$a_{ii} = 0 \quad (i = r-\rho+1,\ r-\rho+2,\ \ldots,\ r). \qquad (14)$$

For this case, in the steady state, we get

$$\Delta\alpha_2(t) = s!\ \eta_s \sum_{k=0}^{\nu} \frac{G_{\nu k}}{(\nu-k)!} t^{\nu-k}, \quad \nu = s-\rho. \qquad (15)$$

The coefficients $G_{\nu k}$ are found from the formulas

$$G_{\nu k} = \frac{1}{k!}\left[\frac{d^k}{dp^k}\frac{\Delta a_\rho(p)}{X(p)}\right]_{p=0}, \quad \Delta a_\rho(p) = \prod_{k=1}^{r-\rho}(p + a_{kk}). \qquad (16)$$

Thus, if the conditions of (14) hold, the degree of the polynominal in t (15), determining the magnitude of the steady-state tracking error, is lower by ρ units than that in (11), which provides for a decrease in the rate of error growth for large values of t. Formula (12) graphically shows how the range-finder parameters and the parameters of the sequence of pulses affect the accuracy of the device's operation.

2. Range Finder with a Simple-Integration Operational Amplifier

By setting $r = 2$ in (1) and (3), we obtain the equations for the range finder with the simplest operational amplifier:

$$(D + a_{11}) U_1(t) = \frac{2K_0 E_s}{\tau_{10} T} \Delta\alpha(t),$$
$$a_{21} U_1(t) + (D + a_{22}) U_2(t) = 0, \qquad (17)$$

$$\Delta\vartheta(t) = \sigma U_2(t), \qquad \Delta\alpha(t) + \Delta\vartheta(t) = \Delta\vartheta_1(t).$$

We shall be interested only in the component $\Delta\alpha_2$. We assume that the translation of the return pulses from the object is described by the function

$$\Delta\vartheta_1(t) = \begin{cases} 0 & \text{for } t \leqslant 0, \\ \eta_0 + \eta_1 t & \text{for } t > 0, \end{cases} \qquad (18)$$

where η_0 and η_1 are constants.

By solving (17) with (18) taken into account, we obtain the expression for $\Delta\alpha_2(t)$:

$$\Delta\alpha_2(t) = \eta_0\left[G_0 + \sum_{k=1}^{2} S_k e^{\mu_k t}\right] + \qquad (19)$$
$$+ \eta_1\left[G_0 t + G_1 + \sum_{k=1}^{2} \frac{S_k}{\mu_k} e^{\mu_k t}\right].$$

The coefficients G_k and S_k are found, respectively, from the formulas

$$G_0 = \frac{a_{11}a_{22}}{X_2}, \quad G_1 = -\frac{2\sigma K_0 E_s a_{21}(a_{11} + a_{22})}{\tau_{10} T X_2^2}, \qquad (20)$$

$$S_k = \frac{\Delta a(\mu_k)}{\mu_k X'(\mu_k)} \quad (k = 1, 2),$$

where

$$X'(\mu_k) = \left\{\frac{d}{dp} X(p)\right\}_{k=\mu_k}. \qquad (21)$$

Here,

$$X(p) = p^2 + X_1 p + X_2, \quad X_1 = a_{11} + a_{22},$$
$$X_2 = a_{11}a_{22} - \frac{2\sigma K_0 E_s}{\tau_{10} T} a_{21}. \qquad (22)$$

The parameters μ_1 and μ_2 are the roots of the characteristic equation

$$X(\mu) = 0.$$

If the range finder is a stable system, then, for the steady state, (19) takes the simple form:

$$\Delta\alpha_2(t) = G_0[\eta_0 + \eta_1 t] + \eta_1 G_1. \qquad (23)$$

The function $\Delta\vartheta_1(t)$ is related to the function $R(t) = R_0 + R_1 t$ which describes the motion of the object with respect to the radar set. This relationship is expressed by

$$\Delta\vartheta_1(t) = \frac{2R(t)}{c},$$

where c is the velocity of propagation of electromagnetic waves.

Consequently, (23) can be rewritten in the form:

$$\Delta\alpha_2(t) = \frac{2G_0}{c}[R_0 + R_1 t] + \frac{2R_1 G_1}{c}. \quad (24)$$

The value of $\Delta\alpha_2$ corresponds to an error in the radius

$$\Delta R(t) = \frac{c\Delta\alpha_2(t)}{2} = G_0[R_0 + R_1 t] + R_1 G_1. \quad (25)$$

If there are very limited possibilities for decreasing the rate of error growth in the range-finder with a simple amplifier, such limitations do not exist in the given scheme. It follows from (19), (20), and (24) that, if a_{22} tends to zero, one can attain an arbitarily small value for the quantity $2G_0R_1/c$ which characterizes the rate of growth of the error. For $a_{22} = 0$, when the operational amplifier becomes a straight integrator, the following relationships hold:

$$X_2 = -\frac{2\sigma K_0 E_s a_{21}}{\tau_{10} T}, \quad G_0 = 0, \quad G_1 = \frac{a_{11}}{X_2},$$

i.e., for the steady state, (24) in the given case takes the form:

$$\Delta\alpha_2 = -\frac{R_1 a_{11}\tau_{10} T}{\sigma K_0 E_s a_{21} c}. \quad (26)$$

To guarantee stability of operation, one must give a positive value to the parameter X_2, i.e., $X_2 > 0$ or $a_{21} > 0$.

According to (26), when $a_{22} = 0$ the tracking error in the steady state is proportional to a_{11}, the parameter of the time separator. The value of this parameter is chosen in accordance with the conditions which guarantee reliable operation of the time separator. As shown by (2), to decrease parameter a_{11} it is necessary to increase both time constants τ_{10} and τ_{11}, which is not always possible. Thus, even if the translations of the return pulses follow (18), the range finder now under consideration has essential disadvantages. In it, the tracking accuracy is limited by the possibilities of decreasing parameter a_{11} of the time separator.

Parameter a_{11} also imposes limitations on the character of the range finder's transient response. In fact, for s = 2, $a_{22} = 0$ and a given value of a_{11}, there remains at our disposal just the one variable parameter a_{21}, which is related to X_2 by the equation

$$X_2 = -\frac{2\sigma K_0 E_s}{\tau_{10} T} a_{21}. \quad (27)$$

In view of the fact that the device is described by a second-order differential equation, it is necessary to have at least two variable parameters. The presence of only one of them entails this, that for $X_2 > a_{11}^2/4$, the roots of the characteristic equation assume complex values, and the tracking system goes into a mode of damped oscillation. With this, the lower the value of a_{11}, the lower the values of X_2 for which this transition occurs. From this point of view, it is well to have a large value of a_{11}, which may be obtained by decreasing the time constants τ_{10} and τ_{11} (and not increasing them, as was required above).

To decrease the steady-state error, it is required to give a large value to the quantity X_2. However, for large values of X_2, the oscillatory process will be characterized by an inadmissibly large overshoot. This is shown by the example, given in the Appendix, of the given range-finder scheme.

Structure of an Operational Amplifier Providing Changes in the Range-Finder's Dynamic Properties

It was previously shown that an increase of the parameter X_2 leads to an oscillatory transient response with a large overshoot. This imposes a limitation on the choice of values for X_2. It also limits the possibilities of decreasing the tracking error in the transient and steady states. We now show that such limitations, and to a large degree, also appear when an operational amplifier with multiple integration is connected in the range finder; due to these limitations, the effectiveness of employing such an amplifier in similar devices is significantly decreased. We now write the characteristic equation of (5) in expanded form. By setting $a_{ij} = 0$ (i = 2,3,..,r) in (5), we get

$$\Delta a(p) = p^{r-1}(p + a_{11}), \quad (28)$$

$$X(p) = p^r + X_1 p^{r-1} + \ldots + X_{r-1} p + X_r = 0. \quad (29)$$

The coefficients X_k are found from the formulas

$$X_1 = a_{11},$$

$$X_k = \frac{2\sigma K_0 E_s}{\tau_{10} T} Z_{k-1} \qquad (k = 2, 3, \ldots, r). \quad (30)$$

In their turn, the quantities $Z_1, Z_2, \ldots, Z_r$ are determined from (6) in terms of the operational amplifier's parameters a_{jk} (j = 2,3,...,r; k = 1,2,...,j).

In the case under consideration, the operational-amplifier implements an s-fold integration. With this, the quantity s is related to r by the equation s = r-1.

The character of the transient response depends on the roots of characteristic equation (29). If it is possible to vary, within wide limits, all the coefficients X_k of (29), this will thereby provide the necessary conditions for changing the character of the tracking system's transient response. These possibilities are easily established, on the basis of (30).

Since the coefficients $X_2, X_3, \ldots, X_r$ depend on the operational amplfier's parameters, they admit variations within quite broad limits. This can not be said, however,

of coefficient $X_1 = a_{11}$, the magnitude of which is determined by the parameter a_{11} of the time separator. As has already been stated, the possibilities of varying this parameter are limited.

On the other hand, the coefficients X_k may be expressed as follows:

$$X_1 = \sum_{k=1}^{r} M_k,$$

$$X_2 = M_1 \sum_{k=2}^{r} M_k + M_2 \sum_{k=3}^{r} M_k + \cdots + M_{r-1}M_r, \quad (31)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$X_r = M_1 M_2 \ldots M_r.$$

Here,

$$M_k = -\mu_k \quad (k = 1,2,\ldots,r) \quad (32)$$

Consequently,

$$a_{11} = \sum_{k=1}^{r} M_k = \sum \mathrm{Re} M_k. \quad (33)$$

Thus, if it is necessary to improve the transient response by increasing the real parts of the roots μ_k then, according to (33), it is necessary to increase the value of parameter a_{11}. In the case of multiple roots,

$$M = M_1 = M_2 = \cdots = M_r$$

Formula (33) takes the simplier form: $a_{11} = rM$, or $a_{11} = -(s+1)M$, i.e., it is required to provide a greater value for the parameter a_{11}, the greater the multiplicity s of integration in the operational amplifier and the greater the necessary magnitude of the quantity M.

The basic disadvantage of the scheme under conlideration is that it is not possible in practice to provide the variation in the time separator's parameter a_{11} which is required for improving the transient response in the automatic range finder. We now show that this disadvantage is eliminated if we use a different operational-amplifier scheme, for example, one for which the processes are described by the equations:

$$(D + a_{11}) U_1(t) = \frac{2K_0E_s}{\tau_{10}T} \Delta\alpha(t),$$

$$a_{21}U_1(t) + (D + a_{22}) U_2(t) = 0,$$

$$a_{31}U_1(t) + a_{32}U_2(t) + (D + a_{33}) U_3(t),$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\sum_{k=1}^{r-1} a_{rk}U_k(t) + (D + a_{rr}) U_r(t) = 0, \quad (34)$$

$$\sum_{k=1}^{r} a_{r+1,k}U_k(t) + U_{r+1}(t) = 0.$$

Equations (34) can be obtained from (1) by adding

$$U_{r+1} = -\sum_{k=1}^{r} a_{r+1,k}U_k,$$

which expresses the simple amplification of the signals U_k with gains proportional to the coefficients $a_{r+1,k}$, with summation of the results. For (34) we find

$$\Delta a(D) = \prod_{k=1}^{r} (D + a_{kk}), A_{r+1,1}(D) = \quad (35)$$

$$= Z_0'D^{r-1} + Z_1'D^{r-2} + \cdots + Z_{r-2}'D + Z_{r-1}'.$$

The equations for the controlled delay circuit and the tracking error when the operational amplifier described by (34) is connected in the device are written in the form

$$\Delta\vartheta(t) = \sigma U_{r+1}(t), \quad \Delta\alpha(t) + \Delta\vartheta(t) = \Delta\vartheta_1(t). \quad (36)$$

Taking the Laplace transforms of (34) and (36), and solving them for $L\{\Delta\alpha(t)\}$, we obtain

$$L\{\Delta\alpha_1(t)\} = -\sigma \sum_{k=1}^{r} \frac{A_{r+1,k}(p)}{X(p)} U_k(0), \quad (37)$$

$$L\{\Delta\alpha_2(t)\} = \frac{\Delta a(p)}{X(p)} L\{\Delta\vartheta_1(t)\},$$

$$\Delta\alpha_1 + \Delta\alpha_2 = \Delta\alpha.$$

For $a_{ii} = 0$ $(i = 2,3,\ldots,r)$, we have

$$\Delta a(p) = p^{r-1}(p + a_{11}),$$

$$X(p) = p^r + X_1p^{r-1} + X_2p^{r-2} + \cdots + X_r, \quad (38)$$

$$X_1 = a_{11} + \frac{2\sigma K_0E_s}{\tau_{10}T} Z_0',$$

$$X_k = \frac{2\sigma K_0E_c}{\tau_{10}T} Z_{k-1}' \quad (k = 2,3,\ldots,r).$$

Here, all the coefficients $X_1, X_2, \ldots, X_r$ depend on the operational amplifier's parameters. Consequently, it is not required to vary the parameter a_{11} in changing from one mode of operation to another, since this transition may be effected by a corresponding change of the operational amplifier's parameters. When an operational amplifier of the type described by (34) is used, the parameter a_{11} is not profitably increased, as was required in the examples given earlier, but, conversely, should be decreased, since the steady-state error is the lower, the smaller the value of a_{11}.

4. Automatic Range Finder with Double Integration Operational Amplifier

In using an operational amplifier with a double integration, the processes of which are described by (34), the automatic range finder's equations are written in the form

$$(D + a_{11}) U_1(t) = \frac{2K_0E_s}{\tau_{10}T} \Delta\alpha(t).$$

$$a_{21}U_1(t) + (D + a_{22}) U_2(t) = 0,$$

$$a_{31}U_1(t) + a_{32}U_2(t) + (D + a_{33}) U_3(t) = 0, \quad (39)$$

$$a_{41}U_1(t) + a_{42}U_2(t) + a_{43}U_3(t) + U_4(t) = 0,$$

$$\Delta\vartheta(t) = \sigma U_4(t),$$

$$\Delta\alpha(t) + \Delta\vartheta(t) = \Delta\vartheta_1(t).$$

We shall be interested below only in the component $\Delta\alpha_2$. Taking the Laplace transform of (39), we get

$$L\{\Delta\alpha_2(t)\} = \frac{\Delta a(p)}{X(p)} L\{\Delta\vartheta_1(t)\}. \quad (40)$$

Here, $\Delta a(p) = (p + a_{11})(p + a_{22})(p + a_{33})$,

$$X(p) = p^3 + X_1 p^2 + X_2 p + X_3,$$

$$X_1 = a_{11} + a_{22} + a_{33} + \varepsilon Z_0',$$

$$X_2 = a_{11}(a_{22} + a_{33}) + a_{22}a_{33} + \varepsilon Z_1', \quad (41)$$

$$X_3 = a_{11}a_{22}a_{33} + \varepsilon Z_2',$$

$$\varepsilon = \frac{2\sigma_0 K_0 E_c}{\tau_{10} T}.$$

The coefficients Z'_0, Z'_1 and Z'_2 are found from the equations

$$Z_0' = -a_{41},$$

$$Z_1 = a_{21}a_{42} + a_{31}a_{43} - a_{41}(a_{22} + a_{33}), \quad (42)$$

$$Z_2' = a_{31}a_{22}a_{43} + a_{21}(a_{33}a_{42} - a_{32}a_{43}) - a_{41}a_{22}a_{33}.$$

We set

$$\Delta\vartheta_1(t) = W_1 t + \frac{1}{2} W_2 t^2, \quad R(t) = R_1 t + \frac{1}{2} R_2 t^2 \quad (43)$$

or

$$\Delta\vartheta_1(t) = \frac{2R(t)}{c} = \frac{2R_1}{c} t + \frac{R_2}{c} t^2, \quad (44)$$

where c is the velocity of propagation of electromagnetic waves. The function R(t) describes the translation of the object with respect to the radar set.

Going to the time-domain function corresponding to (40) while taking (44) into account, we find, for the steady state,

$$\Delta\alpha_2(t) = \frac{2R_1}{c} f_1(t) + \frac{R_2}{c} f_2(t),$$

$$f_1(t) = G_{10} t + G_{11}, \quad (45)$$

$$f_2(t) = \frac{G_{20}}{2!} t^2 + \frac{G_{21}}{1!} t + G_{22}, \quad (t > t_0),$$

where t_0 is the quantity defining the duration of the transient response.

The coefficients G_{1k} and G_{2k} are found from the formulas

$$G_{ik} = \frac{1}{k!} \left\{ \frac{d^k}{dp^k} \frac{\Delta a(p)}{X(p)} \right\}_{p=0} \quad (i = 1, 2). \quad (46)$$

If we set $a_{22} = a_{33} = 0$, we obtain

$$f_1(t) = 0, \quad f_2 = \frac{2a_{11}}{X_3}. \quad (47)$$

The most favorable conditions for increasing tracking accuracy occur when $a_{22} = a_{33} = 0$, when the operational amplifier acts simply as a double integrator.

Appendix

Example 1. To determine the transient response and the magnitude of the steady-state tracking error for the following automatic range-finder data: pulse-repetition frequency $F = 500\ \text{sec}^{-1}$, duration of the return pulses from the object $\alpha = 2\cdot10^{-6}$ sec, length of the tracking pulses $\alpha_0 = 3\cdot10^{-6}$ sec, $K_0 E_3 = 60$v, $\tau_{10} = 2\cdot10^{-4}$ sec, $\tau_{11} = \infty$, the operational amplifier is an integrator, i.e., $a_{22} = 0$. The object moves relative to the radar set with a constant velocity of $R_1 = 300$m/sec.

In correspondence with (2), we find that

$$a_{11} = \frac{\alpha}{T\tau_{10}} = 5\ \text{sec}^{-1}, \quad T = \frac{1}{F} = 0{,}002\ \text{sec}$$

The computational results are given in the first table:

The table fives the steady-state values of $\Delta\alpha_2$ and ΔR. In addition, we have computed the values of the roots μ_1 and μ_2, from which one may derive a representation of the character of the transient response in the tracking system. The table also gives the values of the quantities

$$\rho_1 T = |\operatorname{Re}\mu_1 T|, \quad \rho_2 T = |\operatorname{Re}\mu_2 T|,$$

$$\varphi T = |\operatorname{Im}\mu_1 T| = |\operatorname{Im}\mu_2 T|, \quad (48)$$

from which one may judge if (I.44) hold, as they do in the example under consideration.

As we can see, for $a_{11} = 5\ \text{sec}^{-1}$, there ensues a damped oscillatory mode when $X_2 > 6.25\ \text{sec}^{-2}$. Figure 1 shows a graph of the transient response for the case when $X_2 = 16.12\ \text{sec}^{-2}$ and $X_2 = 45.73\ \text{sec}^{-2}$. The first case corresponds to the roots $\mu_{1,2} = 2.5 \pm i\pi$, the second to the roots $\mu_{1,2} = 2.5 \pm i2\pi$.

For $X_2 = 16.12\text{sec}^{-2}$, the overshoot reaches approximately 14%; as the result of this, the maximum error reaches 105m and, in the steady-state, 93m. For $X_2 = 45.73\text{sec}^{-2}$, the overshoot reaches a significant size - about 69%. Transient responses with such overshoots are inadmissible.

We note in conclusion that the tracking error must not exceed $|\Delta\alpha| < \alpha_0$. If $|\Delta\alpha| \geq \alpha_0$, the return pulses from the object fall beyond the limits of the tracking pulses, and normal operation of the range finder is disrupted. From this point of view, the mode of operation with $X_2 = 2.25\text{sec}^{-2}$ can not actually exist.

Example 2. To determine the transient response and the steady-state tracking error of an automatic range finder with one integrator for the following data: $F = 500\ \text{sec}^{-1}$, $\alpha = 2\cdot10^{-6}$sec, $\alpha_0 = 3\cdot10^{-6}$sec, $\tau_{10} = 10^{-4}$sec, $\tau_{11} = \infty$; $R_1 = 300$m/sec.

This example differs from the previous one in that the time constant τ_{10} has half the value it previously did. Determining the value of the parameter a_{11}, we find that

$$a_{11} = \frac{F\alpha}{\tau_{10}} = 10\ \text{sec}^{-1},$$

the increased value of a_{11} creating more favorable conditions for improving the transient response. Figure 1 shows the graph of the transient response for $X_2 = 50\text{sec}^{-2}$. For this value of X_2, the overshoot is about 7%, i.e., is significantly less than in the case with $X_2 = 45.73\text{sec}^{-2}$, considered in Example 1.

X_3, sec $^{-3}$	μ_1, sec $^{-1}$	μ_2, sec $^{-1}$	$\rho_1 T$	$\rho_2 T$	φT	$\Delta\alpha_3$, sec	ΔR, м
2,25	−0,5	−4,5	0,001	0,009	0	$4 \cdot 10^{-6}$	600
5,25	−1,5	−3,5	0,003	0,007	0	$1{,}9 \cdot 10^{-6}$	285
6,25	−2,5	−2,5	0,005	0,005	0	$1{,}6 \cdot 10^{-6}$	240
10,25	$-2{,}5+i2$	$-2{,}5-i2$	0,005	0,005	0,004	$0{,}98 \cdot 10^{-6}$	147
15,25	$-2{,}5+i3$	$-2{,}5-i3$	0,005	0,005	0,006	$0{,}66 \cdot 10^{-6}$	100
16,12	$-2{,}5+i\pi$	$-2{,}5-i\pi$	0,005	0,005	0,006	$0{,}62 \cdot 10^{-6}$	93
45,73	$-2{,}5+i2\pi$	$-2{,}5-i2\pi$	0,005	0,005	0,013	$0{,}22 \cdot 10^{-6}$	33

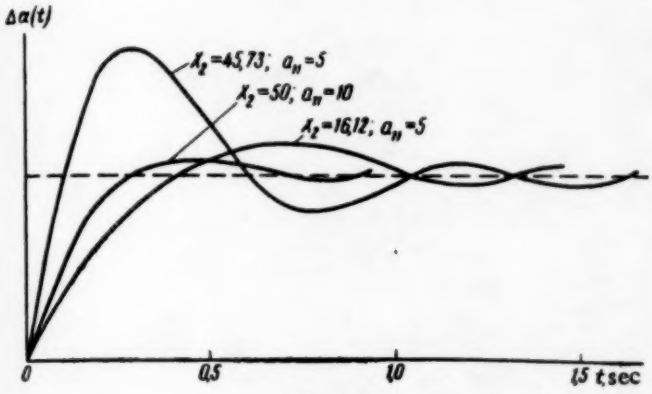

Fig. 1.

Example 3. To determine the parameters of the operational amplifier with a double integrator which provides equal values for all the roots of the characteristic equation, and to find the steady-state tracking error in the region $\Delta\alpha \leq 0.1\,\alpha$ as the object moves relative to the radar set with a constant acceleration of $R_3 = 3g = 30\text{m/sec}^2$. The automatic range-finder data are: $F = 400\text{sec}^{-1}$, $\alpha = 10^{-6}$ sec, $KE_5 = 60$v, $\sigma = 0.5 \cdot 10^{-6}$ sec/v. The computations are carried out for two cases: a) $\tau_{10} = 2 \cdot 10^{-4}$sec, $\tau_{11} = \infty$; b) $\tau_{10} = 2 \cdot 10^{-3}$sec, $\tau_{11} = \infty$.

The steady-state error is determined from the formula

$$\Delta\alpha = \Delta\alpha_3 = \frac{2R_3 a_{11}}{cX_3}. \tag{49}$$

The inequality $|\Delta\alpha| \geq 0.1\,\alpha$ will hold if

$$X_3 \geqslant X_{30} = \frac{2R_3 a_{11}}{0{,}1\,\alpha c} = \frac{2R_3 F}{0{,}1 c \tau_{10}}. \tag{50}$$

In finding the numerical value of X_{30} for cases "a" and "b", we get

$$X_{30} = \begin{cases} 4\,\dfrac{1}{\text{sec}^3} \text{ for } \tau_{10} = 2 \cdot 10^{-4}\,\text{sec} \\ 0{,}4\,\dfrac{1}{\text{sec}^3} \text{ for } \tau_{10} = 2 \cdot 10^{-3}\,\text{sec}. \end{cases}$$

We now determine the operational amplifier's parameters for the value $X_3 = 1.5X_{30}$. For this, we compute the value of ϵ in (41). By substituting numerical values, we get

$$\epsilon = \begin{cases} 120\,\dfrac{1}{\text{sec}} \text{ for } \tau_{10} = 2 \cdot 10^{-4}\,\text{sec} \\ 12\,\dfrac{1}{\text{sec}} \text{ for } \tau_{10} = 2 \cdot 10^{-3}\,\text{sec}. \end{cases}$$

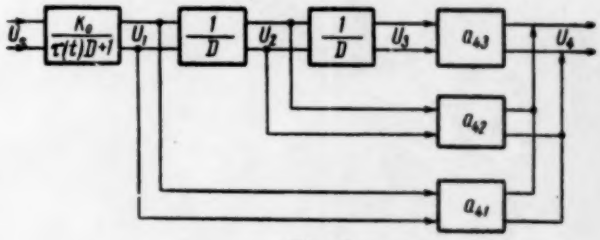

Fig. 2.

τ_{10}, sec	a_{41}	a_{42}	a_{43}
$2 \cdot 10^{-4}$	−0,03	0,08	−0,05
$2 \cdot 10^{-3}$	−0,20	0,18	−0,05

For the case of multiple roots, we have $\mu_1 = \mu_2 = \mu_3 = -M$. For this, the coefficients X_1, X_2, and X_3 are related to M by the equations

$$X_1 = 3M, \quad X_2 = 3M^2, \quad X_3 = M^3. \tag{51}$$

In the given example, the magnitude of M is found from

$$M = \sqrt[3]{15\,X_{30}} \approx \begin{cases} 1{,}82\ \text{sec}^{-1} \text{ for } X_{30} = 4\ \text{sec}^{-3}, \\ 0{,}85\ \text{sec}^{-1} \text{ for } X_{30} = 0{,}4\ \text{sec}^{-3}. \end{cases}$$

On the basis of (41) we have

$$\begin{gathered} a_{11} + a_{22} + a_{33} + \epsilon Z_0' = 3M, \\ a_{11}(a_{22} + a_{33}) + a_{22}a_{33} + \epsilon Z_1' = 3M^2, \\ a_{11}a_{22}a_{33} + \epsilon Z_2' = M^3. \end{gathered} \tag{52}$$

By substituting $a_{22} = a_{33} = 0$ in Eqs. (42) and (52), and then substituting the values of Z'_0, Z'_1, and Z'_2 from (42) and (52), we get

$$\begin{gathered} a_{41} = \frac{a_{11} - 3M}{\epsilon}, \quad a_{31}a_{43} + a_{21}a_{42} = \frac{3M^2}{\epsilon}, \\ a_{21}a_{32}a_{43} = -\frac{1{,}5\,X_{30}}{\epsilon}. \end{gathered} \tag{53}$$

We have at our disposal six variable parameters of the operational amplifier, these being related to M and X_{30} by the three equations of (53). Consequently, three of the six parameters can be given arbitrary values. For example, by setting $a_{21} = a_{32} = 1$, $a_{31} = 0$, we get

$$\begin{gathered} a_{41} = \frac{a_{11} - 3M}{\epsilon}, \quad a_{42} = \frac{3M^2}{\epsilon}, \\ a_{43} = -\frac{1{,}5\,X_{30}}{\epsilon} = -\frac{1{,}5\,R_3}{0{,}1 c K_0 E_5}. \end{gathered} \tag{54}$$

The structure of the time separator and the operational amplifier for the given case reduce to the scheme shown in Fig. 2. This scheme deserves attention, since its technological implementation is comparatively simple.

The results of the computations using (54) are given in the second table.

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DETERMINATION OF THE SECOND-ORDER MOMENTS OF VARIOUS COORDINATES OF AUTOMATIC-CONTROL SYSTEMS BY THE USE OF ELECTRONIC ANALOG COMPUTERS*

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An engineering method is presented for the determination of second-order moments of various coordinates of automatic-control systems by the method of canonical representations of random functions, using electronic analog computers.

In the investigation and design of contemporary automatic-control systems (ACS), an important role is played by the statistical analysis of their dynamic accuracy. The taking into account of random factors is necessary in designing aircraft, automatic-navigation devices, ship-roll dampers, for determining radar-system accuracy and artillery-gear accuracy, for solving information-processing problems, for designing modern drilling equipment, coal-mining machines, automobile suspension systems, etc.

Analytical methods of determing second-order moments of various ACS variables [1-3], due to the large volume of complicated analytical computations and transformations they entail, can be used in engineering calculations only for solving the most elementary problems.

The method of physical simulation of random processes, using electronic analog computers for ACS statistical analysis [3] can be used only for the solution of a narrow class of stationary problems.

The methods of mathematical simulation of random processes, using electronic computers [2, 3], although they theoretically permit one to solve stationary and nonstationary problems of ACS statistical analysis, are very cumbersome to use in practice because of the large amount of complicated analytical computation and transformation which must be carried out in the preparation of the solution as well as in the solution process itself.

For the creation of a practical method of computing second-order moments of various ACS coordinates, it is advantageous to use the canonical representation of random functions [1, 6]

$$X(t) = \sum_{\nu=1}^{\infty} V_\nu x_\nu(t), \qquad (1)$$

where the V_ν are the random, mutually uncorrelated coefficients of the canonical expansion, with dispersions D_ν, and the $x_\nu(t)$ are the canonical expansion's nonrandom coordinate functions.

Thanks to the introduction of the concept of normalized coordinate functions

$$x_\nu^*(t) = D_\nu^{\frac{1}{2}} x_\nu(t) \qquad (2)$$

one can obtain expressions for the second-order moments of various ACS coordinates which are readily amenable to computation by uncomplicated special-purpose electronic computers. We now write these expressions:

a) for the output dispersion

$$D_y(t) = \sum_{\nu=1}^{\infty} y_\nu^{*2}(t); \qquad (3)$$

b) for the output correlation (autocorrelation) function

$$K_y(t, \tau) = \sum_{\nu=1}^{\infty} y_\nu^*(t)\, y_\nu^*(\tau); \qquad (4)$$

c) for the cross-correlation function of the input and output variables at a given moment of time

$$D_{xy}(t) = \sum_{\nu=1}^{\infty} x_\nu^*(t)\, y_\nu^*(t); \qquad (5)$$

d) for the cross-correlation function of the input and output variables

$$K_{xy}(t, \tau) = \sum_{\nu=1}^{\infty} x_\nu^*(t)\, y_\nu^*(\tau). \qquad (6)$$

Here, $y_\nu^*(t)$ is the solution of the differential equation

$$L(p, t)\, y_\nu^*(t) = M(p, t)\, x_\nu^*(t) \quad (\nu = 1, 2, \ldots), \qquad (7)$$

and the expressions

$$L(p, t) = \sum_{i=0}^{n} a_i p^i, \quad M(p, t) = \sum_{j=0}^{m} b_j p^j,$$

$$p \equiv \frac{d}{dt}, \quad m < n,$$

are the operators of the differential equation which describes the automatic-control system being investigated.

By using Relationships (3) - (6), one can construct a computer for the statistical analysis of an ACS which makes it possible to determine the second-order moments

*This paper was given at the All-Union Conference on Computational Mathematics and Computational Technology, at Moscow in 1959

†On the computation of normalized coordinate functions, Cf. [6].

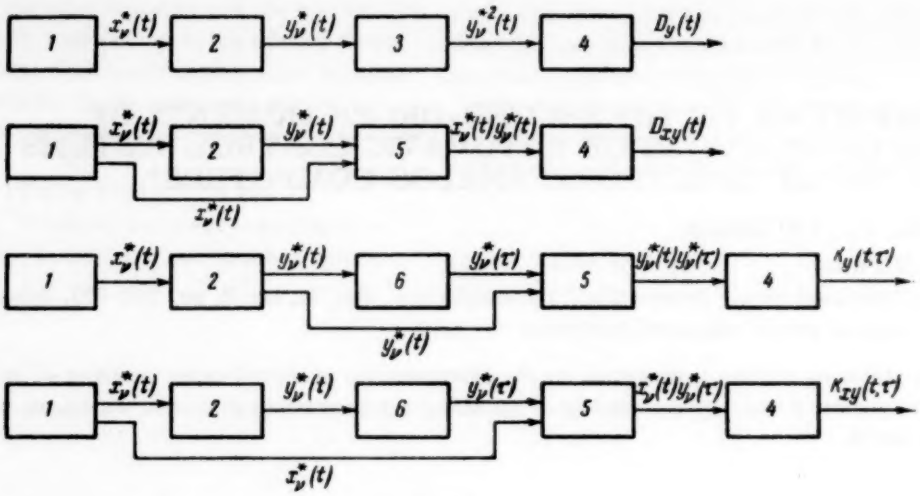

Fig. 1

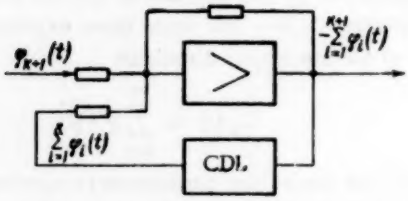

Fig. 2 Principal diagram of special adder.

of various coordinates of the ACS as it is acted on by random disturbances, basing the calculations solely on regular signals, namely, normalized coordinate functions. † Such a device is a special-purpose automatic electronic analog computer. Figure 1 shows the block diagram for a computer designed to determine second-order moments. It contains the following basic blocks: 1 is the block for the memory and for the program control of the computation, 2 is the electronic model of the ACS being investigated, 3 is a squaring device, 4 is a specialized adder, 5 is a multiplier, 6 is a device for introducing delays. The devices used in the computer for squaring, multiplying and introducing delays are standard functional blocks used in electronic simulation [4]. The memory and control block allows input of information (normalized coordinate functions $x_\nu^*(t)$ and the program for controlling the computation) which is written on magnetic tape. For solving (7), the electronic model of the investigated ACS is used. Summation of the series is implemented by means of a special adder (Fig. 2), which is a dc amplifier, operating as a scale amplifier, with additional feedback in the form of a controlled delay line (CDL).

It takes several minutes, on the average, to compute the output dispersion or the cross-correlation function of the ACS's input and output variables on such a machine. It takes some tens of minutes to determine the output autocorrelation function and the ACS's output variable.

Example. As an example, we consider the problem of determining the dispersion of the output coordinate ψ of an ACS during the transient response, when random disturbances act on it. The equations of the system, consisting of object plus controller, are written in the form:

$$\begin{aligned} \omega &= \dot{\psi}, \\ \dot{\omega} &= a_1\omega - a_2\beta - a_3\delta + X(t) \\ \dot{\beta} &= \omega + a_4\beta + a_5\delta, \\ \dot{\delta} &= \frac{1}{T}[i(\psi + \mu\omega)] - \delta. \end{aligned} \tag{8}$$

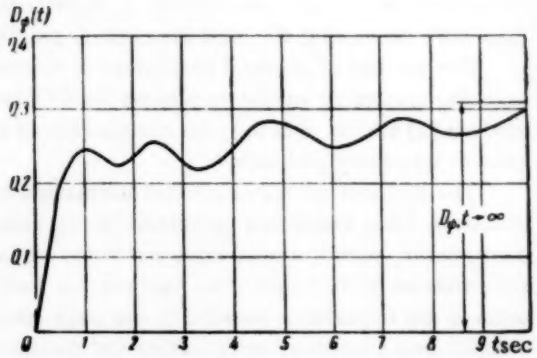

Fig. 3. Curve of the dispersion of the output coordinate ψ of the ACS described by differential equations (8).

Calculation of the dispersion $D_\psi(t)$ of the ACS's output coordinate ψ for $0 \le t \le T = 10$ sec, when the random disturbance X(t) with correlation function $K_X(t,\tau) = \exp(-|t-\tau|)$ acts on it, was implemented in accordance with the block schematic (the first line thereof) shown on Fig. 1. With this, ten terms of the canonical representation of the random disturbance X(t) [6] were taken into account. The time for the computation was 110 sec. Figure 3 gives the curve for the dispersion $D_\psi(t)$.

What has been presented here shows that the use of canonical representations of random disturbances, electronic models of the system being investigated, and additional uncomplicated electronic computing hardware make

it possible to find, quite simply from the point of view of engineering practice, the second-order moments of various coordinates of a broad class of automatic-control systems.

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FINITE AUTOMATA. I

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The views of the authors are presented on the basic content and problem areas of the theory of finite automata and on the interrelationships of this theory with the theory of relay-contact circuits; also considered are the possible ways of realizing finite automata.

INTRODUCTION

As soon as it became clear that the propositional calculus was adequate for the technical problems related to the investigations of many relay (switching) devices [1-3], the abstract concept of the "nonsequential switching circuit" was formulated. For many switching circuits, such an abstraction turned out to be natural, and the propositional calculus became the basic mathematical apparatus for analyzing and synthesizing within the limits of this abstraction. However, this abstraction turned out to be inadequate, even for many important switching circuits, since it gave, in principle, no possibility of considering circuit operation in time, and only by artificial methods can be used for solving certain problems in the theory of such circuits.

The desire to introduce time ("sequentiality") into consideration explicitly, and to find an adequate mathematical apparatus for the description of circuits operating through time went beyond the limits of application of solely algebraic, logical operations (the propositional calculus).

"Motion" in such devices was given by special formulas and tables (Cf., for example, [3-7]), by geometric forms (motion of a representative point along the vertices of multidimensional cubes, or along the nodes of some graph or another) (Cf., for example, 4,6-9]) and, finally, by integer recursive functions (Cf., for example, [9-14]). In these works, account was taken, to a greater or lesser degree, of the peculiar features of some concrete method or other of technically realizing logical operations (in particular, the specific features of relay contact technology), i.e., essentially, the soil was prepared for the introduction of new, broader abstractions. It gradually became understood that what was at issue was the isolating of a specific class of dynamic systems*; and that systems constructed of elements having, not two, but an arbitrary, though finite, number of possible states had to be encompassed by these new abstractions.

While, in ordinary dynamic systems, the space and time coordinates have continua for their domains, the basic idiosyncrasy of this special class of dynamic systems is that their time has a countable set for its domain, while their coordinates have finite sets for theirs.

As these abstractions were developed, various names were given to them: logical nets [10], generalized nerve nets [15, 16], finite automata [16], nonprimitive circuits [14], sequential machines [18, 19], robots [17] etc.

The authors prefer the name finite automata for these abstractions.

Certainly, not all switching devices (i.e., those exhibiting relay functioning) are encompassed by these abstractions. For example, the abstraction "finite automata" is not adequate for systems containing devices in which the coordinates are chosen from finite sets, while the time has a continuum for its domain, although these are relay devices. On the other hand, the abstraction "finite automata" corresponds organically to all relay contact circuits which are devoid of such devices. But with certain natural idealizations, many other important technical devices also correspond to it, for example, machine-stand automata, mass-production lines, various digital machines, automata for piecework production, certain abstract models of nervous systems (for example, the nerve nets of McCulloch and Pitts), etc.

2. Basic Definitions and Terminology

As was said earlier, a finite automaton is a special type of dynamic system. The first characteristic of this class of dynamic system is that, in describing its actions, only discrete moments of time are of importance, the intervals between these moments (which, in general, are of unequal duration) being called cycles of the finite automaton's operation. Let p be one of these moments ("the present moment"), and let p-1, p-2,...,2,1,0 be the series of preceding moments of time. In this notation, 0 corresponds to the initial moment of automaton operation. We assume that some conditions or other are given

* In this sense, the older abstraction - the nonsequential relay contact circuit - corresponded to a static system.

which unambiguously define the decomposition of the time axis into cycles†.

In the general case, the dynamic systems under consideration are subject to external stimuli. In the sequel, these stimuli will be called inputs.

Definition. A finite automaton is a dynamic system which, at the discrete moments of time under consideration, satisfies the following conditions.

1. At each of these moments of time, the system can be in just one of a possible finite number k of states.

2. At these moments of time, the system's inputs are also restricted to be chosen from a finite number r of possible states.

3. At any of the moments of time considered, the state of the system is uniquely defined by the state of the system and the state of the input at the previous moment of time.

To distinguish one automaton state from another, we assign a different symbol to each of the k possible states. We denote these symbols by $\kappa_1, \kappa_2, \ldots, \kappa_k$. We call the set of these symbols, denoted by $\{\kappa\}$, the alphabet of the states. Analogously, we write the symbols $\rho_1, \rho_2, \ldots, \rho_r$ for the input states. They form the automaton's input alphabet $\{\rho\}$‡.

We denote by $\kappa(t)$ and $\rho(t)$ the symbols for the automaton state and, input respectively, at time t. Then, by virtue of point 3 in the definition of the finite its operation is described by the recursion formula

$$\kappa(p) = F[\kappa(p-1),\ \rho(p-1)], \qquad (1)$$

where F is a single-valued function which puts a definite symbol from the state alphabet into correspondence with each pair of symbols, one of which is chosen from the state alphabet, the other from the input alphabet.

Thus, the abstraction introduced by the concept of "finite automata" singles out a class of systems whose processes are described, not by differential equations, nor yet by difference equations of general form, but by specific equations of the type given in (1). The theory of finite automata concerns the general properties of such a type of system. Therefore, from the point of view of this theory, all systems are identical whose descriptions, with the applicable idealizations, reduce to an equation of type (1) with the same function F in the right member. Moreover, automata described by equations of type (1) with different functions F in their right members can still turn out to be equivalent, in a definite sense, to one another (Cf. section 4 below).

In the actual systems subsumed under the heading of "finite automata," a change of state of an automaton can, in some cases, be taken directly as being the result of the system's operation. In other cases, the automaton has an "output," whose state is uniquely determined by the automaton's state, but is not identical with it**.

Of course, in a finite automaton, the number of possible output states may, but in general does not, coincide with k. Let the output have l ($l \leq k$) states. We write each of the possible output states by a symbol $\lambda\{\lambda_1, \lambda_2, \ldots, \lambda_l\}$, which we call the output alphabet. Then, to describe the operation of an automaton with outputs, (1) must be supplemented by the "instantaneous" relationship

$$\lambda(p) = \Phi[\kappa(p)], \qquad (2)$$

which puts a symbol from the output alphabet into correspondence with each symbol from the state alphabet.

Equation (1) can be given by Table 1, which we call the fundamental table of the finite automaton.

TABLE 1

	κ_1	κ_2	...	κ_k
ρ_1	κ_3	κ_5	...	κ_3
ρ_2	κ_1	κ_k	...	κ_6
...	...	...	...	...
ρ_r	κ_4	κ_5	...	κ_2

This table is set up in the following way: a cell of the table is defined by giving one symbol each from the $\{\rho\}$ and $\{\kappa\}$ alphabets; by taking these symbols to represent matters at time p-1, i.e., by considering them to represent $\rho(p-1)$ and $\kappa(p-1)$, we determine $\kappa(p)$ from (1); we assign the value of $\kappa(p)$ thus determined to this cell.

If the automaton has outputs, then (2) defines the table of correspondences between κ and λ (Table 2).

TABLE 2

κ	κ_1	κ_2	κ_3	κ_4	κ_5	...	κ_k
λ	λ_2	λ_4	λ_1	λ_3	λ_l	...	λ_5

† It may be assumed, for example, that the continuous time axis is previously divided in some manner into cycles which depend neither on the automaton's operation nor on its external stimuli. In other cases, the moment when a new cycle begins can be taken as the moment when some change in the automaton's input occurs. Finally, the cycles may be defined in such manner that the beginning of a new cycle will also depend on the state of the automaton, or on its output (Cf. part II of the present paper, to be published in the next number of this journal).

‡ In particular, one can number the automata and input states. Then, the state and input alphabets would be comprised of sets of numbers, 1,2,...,k and 1,2,...,r. Such alphabets might be convenient in investigations of individual automata. They are inconvenient in those cases when automata are joined, one to another, since the same symbol here, say the number 3, would apply to states and inputs of different automata.

** For example, in an automaton used for sending alarm signals, the important thing is not its state at each moment, but only whether this state does or does not correspond to an alarm signal.

Together with the finite automaton's fundamental table (Table 1), we shall consider an infinite tape (Table 3), characterizing, for a given initial state and a given sequence of input symbols, the sequence of appearance of state symbols as determined by (1) (or, what amounts to the same thing, as determined by Table 1).

TABLE 3

t	0	1	2	3	4	...
ρ	ρ_1	ρ_3	ρ_5	ρ_7	ρ_2	...
$\varkappa$	$\varkappa_4$	$\varkappa_1$	$\varkappa_k$	$\varkappa_5$	$\varkappa_1$	...

On this tape, each moment of time t is put into correspondence with a ρ and a κ. We call it the state tape.

A triple of symbols, $\kappa(p-1)$, $\rho(p-1)$ and (p), such as shown on Table 3 by the heavy lines, and chosen for any p, is related internally by (1). We call such a triple a triad.

TABLE 4

t	1	2	3	4	5	...
ρ	ρ_1	ρ_3	ρ_5	ρ_7	ρ_2	...
λ	λ_2	λ_3	λ_5	λ_1	λ_2	...

In cases when the system is defined by (1) and (2) (or by Tables 1 and 2), we shall also consider the automaton's output tape (Table 4), in which each moment of time t is put into correspondence with a pair of symbols ρ and λ.

We take particular note of the case when the input state does not change with time - automata of such type are called autonomous. For an autonomous automaton, (1) has the form:

$$\varkappa(p) = F[\varkappa(p-1),\ \rho(0)], \qquad (3)$$

where $\rho(0)$ is a time-invariant symbol which can be considered as a parameter.

The operation of an autonomous automaton can be illustrated by a graph.

We place k points on a plane, assigning to each point one symbol corresponding to one of the automaton's possible states. Equation (3) then defines the sequence of changes of state, i.e., the directed transitions from point to point. These directions can be indicated by arrows. It is easily seen that, in general, several arrows can lead to the same point. From a given point, however, at most one, and possible no arrows may lead.

The set of k points, together with the arrows as just described, is called the graph of the autonomous finite automaton.

In a nonautonomous automaton, there can be r input states. Therefore, by fixing and by leaving thenceforth invariant the input state, one can organize r different autonomous automata from a given nonautonomous automaton, each of the autonomous automata having its own graph. The set of these r graphs completely characterizes the nonautonomous automaton. Figure 1 shows an example of such a set, comprised of three graphs; a state table (Table 5) corresponds to this automaton.

Fig. 1

TABLE 5

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
ρ_1	x_1	x_4	x_4	x_4	x_3	x_5	x_7
ρ_2	x_1	x_1	x_2	x_1	x_3	x_4	x_6
ρ_3	x_4	x_1	x_2	x_3	x_7	x_6	x_6

3. The General Problems Arising in Connection with the Introduction of the Abstraction "Finite Automata"

Although the abstraction of "finite automata," as this term was introduced in section 2, is a far-reaching one, there arose with its introduction [i.e., with the isolating of the class of systems described by (1), or by (1) + (2)] a number of problems, equally important for all actual systems for which this concept is adequate.

One of these problems, that of the representation of events in finite automata, was solved (inefficiently, it is true) by Kleene's theorems [16] (Cf. also [19-21]).

This is not the sole problem, and a number of important problems in finite automata theory are today not only unsolved, but not even rigorously formulated. These problems arise principally in connection with the fact that both the state tape and the automaton's output tape are infinite. It is therefore meaningless to say that these tapes are "given." All that may be given are algorithms which permit, in a finite number of steps, choosing of the symbol from given alphabets which must be written in an arbitrary cell of the tape.

We denote by A_ρ and A_κ the algorithms corresponding to the upper (ρ) and lower (κ) rows of the state tape, and by A_ρ and A_λ the analogous algorithms for the output tape. From this point of view, a finite automaton, defined by (1) or by (1)+(2) (i.e., by Table 1 or Tables 1 and 2), is essentially an operator which establishes the relationships between algorithms A_ρ and A_κ, or between A_ρ and A_λ.

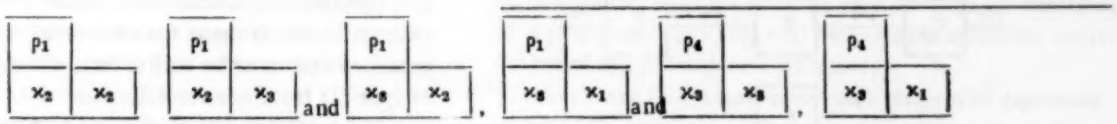
In connection with this, no scientific problems arise in the task of analysis, i.e., in the task of determining,

for a given automaton, the algorithm A_κ (or A_λ) from A_ρ and conversely, and here one may only speak of developing more or less convenient ways of implementing the operations defined by the "finite automaton" operators.

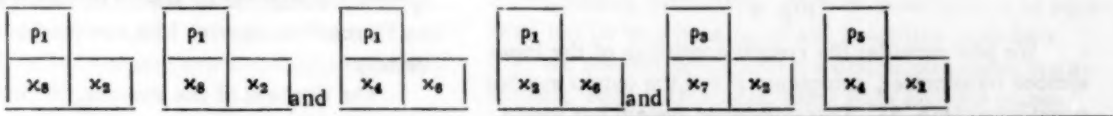
Matters are otherwise when the tape algorithms are given (i.e., A_ρ and A_κ, or A_ρ and A_λ) and it is required to synthesize the automaton (i.e., to construct (1)).

We first consider the case when it is required to synthesize the automaton from the given algorithms A_ρ and A_κ. It is clear that this problem cannot always be solved, since the tape defined by algorithms A_ρ and A_χ may contain triads which are mutually contradictory.

We call two triads contradictory if, in these triads, the symbols $\rho(p-1)$ and $\kappa(p-1)$ are identical, but the symbols for $\kappa(p)$ are different. For example, the pairs of triads



are contradictory, while the pairs of triads



are noncontradictory.

Each cell of Table I contains one symbol. Therefore, the existence of contradictory triads is incompatible with Table 1, i.e., it contradicts the single-valuedness of the function F in (1).

Thus arises the first problem: knowing the algorithms A_ρ and A_κ, to find an algorithm for recognizing their "noncontradictoriness," i.e., the absence of contradictory triads on the state tape.

This problem may be formulated precisely, as follows. We are given the recursive functions $\kappa(p)$ and $\rho(p)$, each of which can assume a finite number of values. It is required to construct a recursive functional $\Psi[\kappa(p), \rho(p)]$ (Cf. [23] Chap. 9) which equals unity on those functions $\kappa(p)$ and $\rho(p)$ for which there exists the function $F(u,v)$, defined on the finite set constituting the domain of u and v such that, for any p, the relationship $\kappa(p) = F[\kappa(p-1), \rho(p-1)]$ holds, and which equals zero otherwise. If, as may be assumed, the problem as formulated is algorithmically unsolvable, the necessity then arises of posing the more restricted tasks of determining a recursive functional Ψ on the functions $\kappa(p)$ and $\rho(p)$ which lie in a more restricted class than the class of all the recursive functions which assume the given number of values.

Although the automaton's state tape is infinite, the numbers r and k are finite, so that the total number of different triads on the tape is also finite. After the question of the "noncontradictoriness" of the algorithms is resolved, there arises a second problem: for given "noncontradictory" algorithms A_ρ and A_κ, to find an algorithm for seeking all the different triads contained on an infinite state tape. The solution of this problem would immediately permit one to fill in all or some of the cells of Table 1, and to define the automaton with an exactitude to within the cells which remained unfilled and could be filled in arbitrarily.

In the case when a finite number of state tapes is given, and it is required to find an automaton which reproduces all these tapes, no new problem arises, since the algorithm for observing contradictions and the algorithm for determining all the different triads may be applied to each tape in turn. New problems arise only when the number of state tapes is infinite.

The matter is significantly more complicated when it is required to synthesize an automaton from its output tape. Here the additional problem arises: knowing the algorithms A_ρ and A_λ, to determine k, i.e., the number of states which must be realized in the automaton, and to find the function Φ in (2), i.e., from the given output tape to construct a noncontradictory state tape.

It is precisely problems of this type which are most interesting in practice, since the requirements on the automaton to be synthesized are ordinarily given directly in "input-output" terms.

It is necessary to mention that the general problem of synthesizing finite automata from given operating conditions has thus far not been precisely formulated. The difficulty of a rigorous posing of this problem stems from the following. In the majority of questions arising in practice, the automaton must transform a given infinite class of input sequences into a required class of output sequences. Up to now, however, there has been developed no sufficiently general and convenient mathematical apparatus in terms of which an adequate description of these infinite classes of sequences could be given. In the present case, a description in terms of sets of tables [15] is unsatisfactory, since the original (verbal) given conditions of operation are posed in other ("logical") terms (closer to the language developed in [22]), while no algorithm has been established for making the transition from the language of sets of tables to the "logical" language.

All algorithmic problems of this nature vanish, and the synthesis problem becomes much easier, in the case when the automaton's tapes are finite.

4. Mappings of Finite Automata and Their Equivalence

In considering an automaton with outputs, we previously used the "instantaneous" relationship of (2).

This relationship can be considered as a rule for transforming state symbols to output symbols. An abstract device which realizes (2) is called a symbol transformer. There is, of course, an analogous symbol transformer at the automaton's input; it will transform the symbols of some external stimulus alphabet to symbols of the automaton's input alphabet.

We now assume that two automata are given: automaton A, operating with input alphabet $\{\alpha\}$ and with state alphabet $\{a\}$, and automaton B, with input alphabet $\{\beta\}$ and state alphabet $\{b\}$. We further assume that there are two symbol transformers: one of them (the input transformer) transforms the symbols $\{\alpha\}$ to $\{\beta\}$, and the second (output) transforms symbols $\{b\}$ to $\{a\}$.

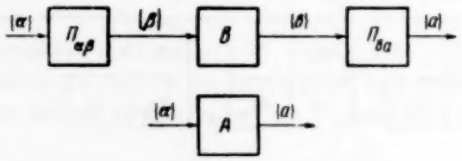


Fig. 2

We now consider the system consisting of the input symbol transformer, automaton B and the output symbol transformer (Fig. 2). This combined system has inputs from the alphabet $\{\alpha\}$ and outputs from the alphabet $\{a\}$, the same as automaton A††.

If we can choose input and output symbol transformers such that, for any sequence of input symbols from alphabet $\{\alpha\}$ and for any initial state of automaton, A, there would exist an initial state of automaton B such that there would appear, at the output of the combined system (automaton B and the two transformers), the same sequence of symbols from the alphabet $\{a\}$ as the sequence of state symbols in automaton A, then we shall say that automaton B is a mapping of automaton A (or that automaton A is mapped onto automaton B).

We symbolize this fact by:

$$A \subseteq B \quad \text{or} \quad B \supseteq A.$$

If simultaneously

$$A \subseteq B \quad \text{and} \quad A \supseteq B,$$

then the automata A and B will be said to be equivalent, and we symbolize this circumstance by $A \asymp B$.

We now give a precise definition of the concept of mapping.

The automaton A ($\{a\}$, $\{\alpha\}$) is mapped onto automaton B ($\{b\}$, $\{\beta\}$) (and automaton B is a mapping of automaton A) if, to each pair (a_i, α_j) of symbols from the alphabets $\{a\} = \{a_1, .., a_k\}$ and $\{\alpha\} = \{\alpha_1, .., \alpha_r\}$, one can set up a correspondence of such a pair ($\{b_i\}$, $\{\beta_j\}$) of sets of symbols from the alphabets $\{b\} = \{b_1, .. b_n\}$ and $\{\beta\} = \{\beta_1, .., \beta_m\}$ such that, if

$$F_A(a_i, \alpha_j) = a_s, \qquad (4)$$

then, for any symbols $b_{q_i} \in \{b_i\}$ and $\beta_{p_j} \in \{\beta_j\}$ the following condition holds

(5)

Here, F_A and F_B are the functions in the left bers of equations of the type of (1) for automata A and B respectively.

Two equivalent automata do not differ practically from one another – their differences are external, consisting only in the methods chosen for coding the automata states and inputs. In spite of this, two equivalent automata can be described by completely different equations of type (1). Even the variables in these equations may be taken from different alphabets. Thus, for example, two automata with identical numbers of states and inputs, differing only by the notation of the states and the inputs, i.e., the alphabets from which the symbols are taken and the manner in which these are associated with states and inputs, are known to be equivalent, although the equations of type (1) for them are different.

If $A \subseteq B$, but $A \not\asymp B$, this means that there exists a symbol transformer by means of which automaton B can be "forced" to operate like automaton A, but not conversely.

The concept of the mapping of finite automata, and of their equivalence, is closely related to the topological peculiarities of their graphs.

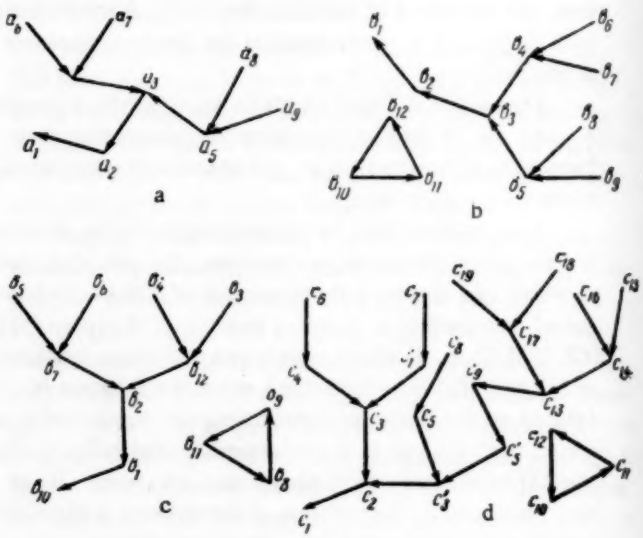


Fig. 3

As an example, we consider the graphs of the four autonomous finite automata, given in Fig. 3. The automata of Fig. 3, b, and c are equivalent, since they differ only by the nomenclature of the states. The automaton of Fig. 3a, is mapped onto each of the automata of Fig. 3, b, and c. This becomes clear by supposing that a trans-

†† To be sure, such a combined system might not be a finite automaton, since its output symbol at time p might not be uniquely determined by its output and input symbols at time p-1. We shall assume that the combined system under consideration is a finite automaton.

former is placed at the output of the automaton of Fig. 3,b which places each symbol b_i ($i = 1,\ldots,8$) into correspondence with any of the a_i ($i = 1,\ldots,8$).

All three of the automata thus far considered are mapped onto the automaton of Fig. 3,d. The output transformer which it is necessary to append to the automaton of Fig. 3,d for it to operate the same as the automaton of Fig. 3,b must place each of the symbols c_i into correspondence with b_i for i = 1,2,6,7,8,9,10,11, and 12. Symbols c_3 and c'_3 are made to correspond with the same symbol b_3, symbols c_4 and c'_4 with b_4 and symbols c_5 and c'_5 with b_5. In addition, symbols $c_{13}, c_{14}, \ldots, c_{19}$ are placed into correspondence with any of the symbols $b_1, \ldots, b_{12}$.

The problem now arises: knowing the fundamental tables of two automata, or their equations of type (1), to determine whether these automata are equivalent, or whether one of them is mapped onto the other‡‡. In other words, it is required to find criteria for the mappings and equivalences of automata.

The concept of equivalence of automata, and of their mappings, allows one to introduce the concept of the abstract aggregating (assembly) of finite automata, i.e., of assembling a finite automaton from standard elementary automata.

5. Abstract Assembly of a Finite Automaton

We now consider a device with s input wires $u_1, \ldots, u_s$ and n generalized coordinates $x_1, \ldots, x_n$. At the times 0,1,2,3,...,p, each input and each coordinate may have only one of a finite number of values, which we shall symbolize by symbols from some alphabets. At any moment of time, each coordinate is determined uniquely by the values of all or some of the inputs and coordinates at the preceding moment of time.

The operation of such a device is described by the recursion relationships

$$x_i(p) = f_i[x_1(p-1), \ldots, x_n(p-1); \; u_1(p-1), \ldots, u_s(p-1)] \; (i = 1, \ldots, n) \qquad (6)$$

or, in vector form,

$$x(p) = f[x(p-1), \; u(p-1)]. \qquad (7)$$

If an x_i can have χ_i different values ($i = 1,\ldots,n$), then the vector $x(p)$ can have $k = \prod_{i=1}^{n} \chi_i$ different values. We call the vector x the state of the device being considered, and we choose its k values from the symbols $\kappa_1, \ldots, \kappa_k$ of the alphabet $\{\kappa\}$. Analogously, the input vector u, with $r = \prod_{j=1}^{s} \xi_j$ values (where ξ_j is the number of values of u_j) is called the input state, and we relate its values to the symbols $\rho_1, \ldots, \rho_r$ of the alphabet $\{\rho\}$. With this notation, (7) reduces to (1), which means that the device just described is a finite automaton.

Equations (6) can also be understood as another form of equation of the type (1). Different equations of type (6) can correspond to one and the same equation of type (1), but they all describe the processes in equivalent automata.

In the definition of a finite automaton, given in section 2, and in the equation of type (1) which corresponded to it, we had to do only with automata states and inputs. Now, in making the transition to equations of type (6), we introduced coordinates and input wires (input lines). We shall call the transition from (1) to the equivalent (6) a choice of structure for the finite automaton, and (6) will be called its abstract structure (AS). A technical device which, with the suitable idealizations, is described by equations of type (6), will be called a technical realization of the automaton's structure.

Let there now be given several systems of equations of type (6), each of which is the abstract structure of a corresponding automaton. Each of these systems of equations has its own coordinate and input-line alphabets.

We now introduce into the discussion symbol transformers, each with several input lines and one output. The symbols from the transformer's output alphabet appear at its output at the same instant that the symbols from their respective alphabets are applied to the input lines.

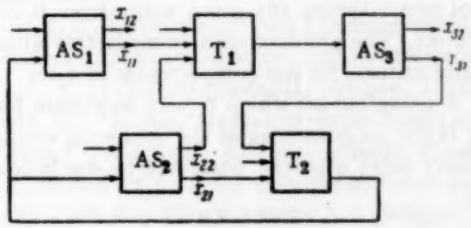


Fig. 4

We use transformers as means for the interconnection of AS (as, for example, on Fig. 4), choosing them in such manner that the corresponding AS alphabets and transformers are properly linked. We call such a set of AS, interconnected via symbol transformers, a net. The coordinates of the net are comprised of all the generalized coordinates x_i of all the AS, including those not applied to any transformer. The net's input lines are both the AS input lines on which no transformer acts, and the transformer input lines on which no AS coordinate acts.

In order to obtain the equations for a net, it is necessary to use the given equations for the transformers for eliminating from the systems of equations of type (6) the corresponding variables for the AS input lines, replacing them by the AS coordinates and the variables of the transformers' input lines. As the result, the net as a whole is also described by a system of recursion relationships of

‡‡ Certainly, what we have in mind here is not the trivial solution, obtained by the exhaustive trial of all meaningful correspondences of symbols.

$x(p-1)$			$u(p-1)$			$\kappa(p-1)$	$\rho(p-1)$	$\kappa(p)$	$x(p)$		
x_1	x_2	x_n	u_1	u_2	u_s				x_1	x_2	x_n

TABLE 6

type (6), i.e., the net itself is the abstract structure of some finite automaton.

From what has been said, it follows that a finite automaton can be constructed from other finite automata by joining them in a net. The question naturally arises as to the possibility of creating a set of automata, or even all meaningful automata, from a small collection of original automata – "elements." Such construction of automata we call abstract assembly. The technological value of this concept is obvious.

The introduction of the concept of abstract assembly leads immediately to the posing of the following problems: to construct algorithms which answer the question as to the possibility of constructing nets from a given set of AS and transformers, these nets being mappings (in the sense of section 4) of given automata, and to specify an algorithm for the efficient implementation of this construction.

A set of AS and transformers from which one can construct nets mapping any given automaton is called a complete set. The problem then immediately arises of determining criteria for the completeness of a set.

A complete set which is very important for technology is the set consisting of the following elements: a) a binary delay element, described by the following equation

$$x(p) = z(p-1)$$

where x and z are given in an alphabet which contains only two symbols (for example, 0 and 1); b) some set of logical elements which permits the realization of any logical function (this set may consist for example, of conjunction, disjunction and negation elements).

From this set, a net which is the mapping of any finite automaton can be constructed. Indeed, let a finite automaton be given by Eq. (1), i.e., let the function F and the alphabets $\{\kappa\}$ and $\{\rho\}$ be given.

We choose two numbers n and s such that the conditions $2^n \geq k$ and $2^s \geq r$ are satisfied. We introduce the variables $x_1,\ldots,x_n$ and $u_1,\ldots,u_r$, where each of these variables assumes only the values 0 and 1.

We consider Table 6. On its left side are enumerated all possible combinations of all values of $x_1,\ldots,x_n$, $u_1,\ldots,u_s$, i.e., Table 6 has 2^{n+s} rows. Each different combination of variables $x_1,\ldots,x_n$ is placed in correspondence with its own symbol from the alphabet $\{\kappa\}$. Since the number of these symbols is $k \leq 2^n$, there may remain combinations $x_1,\ldots,x_n$ for which the alphabet $\{\kappa\}$ cannot supply symbols different from other symbols already used. We assign arbitrary, previously used symbols from $\{\kappa\}$ to these combinations $x_1,\ldots,x_n$.

We proceed analogously with the combinations of variables $u_1,\ldots,u_s$, designating them by symbols from $\{\rho\}$. We write the selected symbols from $\{\kappa\}$ and $\{\rho\}$ in the corresponding row of columns $\kappa(p-1)$ and $\rho(p-1)$ of Table 6.

Thus, to each set of variables $x_1,\ldots,x_n$; $u_1,\ldots u_s$ there corresponds a pair of symbols from the alphabets $\{\kappa\}$ and $\{\rho\}$. By taking these pairs of symbols as standing for $\kappa(p-1)$ and $\rho(p-1)$, we find $\kappa(p)$ from (1), and write these values in the corresponding rows of column $\kappa(p)$ of Table 6. We can then immediately fill up the right side of Table 6, since we have already related a definite combination of values $x_1,\ldots,x_n$ with each symbol from $\{\ \}$ ***

The table thus constructed determined the value of all x_i at time p from the given x_i and u_j at time $p-1$, i.e., it gives n logical recursion relationships of the type of (6):

$$x_i(p) = L_i[x_1(p-1),\ldots,x_n(p-1);\ u_1(p-1),\ldots,u_s(p-1)] \quad (i=1,\ldots,n). \tag{8}$$

For any combination of values of the variables, the values of all the functions L_i are read off from Table 6.

It is easily seen that the obtained equations (8) are the abstract structure of an automaton which maps the original automaton which was given by an equation of type (1).

Each equation in (8) can be realized in the form of an elementary one-cycle lag, with equation

$$x_i(p) = z_i(p-1) \qquad (i=1,\ldots,n), \tag{9}$$

and a logical, instantaneously acting transformer

$$z_i = L_i(x_1,\ldots,x_n;\ u_1,\ldots,u_s) \quad (i=1,\ldots,n), \tag{10}$$

which, in the given case, is also a transformer of the coordinate x_i and input u_j symbols to input z_k symbols of the lag of (9). In the given special case, all the symbols x_i, u_j, and z_k are chosen from the same alphabet, namely, $\{0,1\}$. The entire set of equations (8) is a net composed of n lags of the form (9) and n transformers of the

*** If $k < 2^n$ and if several combinations, $x_1,\ldots,x_n$ were related to one and the same symbol of $\{\kappa\}$, then we choose any one of them in any case when the corresponding symbol arises.

form (10). Since any logical transformer is realized from the set of logical elements given by us in the conditions of the problem, this shows that this set of logical elements, supplemented by a delay element, is a complete set of the elements necessary for constructing any automaton.

It can be shown analogously that a set of elements allowing any function of q-valued logic ($q > 2$) to be reproduced, and supplemented by a delay element of the form of (9), wherein the input and output assume values from a q-symbol alphabet, also comprises a complete set.

In this section we first used the concept of a delay element. This concept requires the following additional elucidation.

When any symbol from the input alphabet is applied to the input of delay element (9), the corresponding symbol from the output alphabet appears at its output after a one-cycle delay. It was stated, at the very beginning of this paper, that the cycles could be nonuniform, and that the division of the time axis into cycles was uniquely determined by some additional conditions. So long as we had to do with abstract structure and abstract assembly, i.e., essentially with the replacement of (1) by equations of the form (6), this circumstance was of no significance since in (1) and (6) the sequentiality (i.e., the character of the variation of the argument p) was considered as given. In using the complete sets, described above, for the technological realization of automata, it is assumed that the automata contain special devices – sequencing transducers which, in correspondence with given conditions, process the signals for cycle beginning and apply them to delay elements. This stimuli of the delay elements are not external in the sense in which the term "external stimuli" was used above; they are not encoded by symbols from $\{\rho\}$ in setting up (1), and are not germane to the stimuli u_j in (6). Therefore, even an autonomous automaton may contain these stimuli, which are essential only for the technological realization, and are not taken into consideration when one deals with abstract structure or abstract assembly.

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A UNIVERSAL FUNCTION GENERATOR BASED ON THE PRINCIPLE OF QUADRATIC APPROXIMATIONS

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Methods are presented for quadratic approximations for functions given either analytically or graphically. Simple formulas and relationships are derived for finding the rule for argument partitioning. For obtaining a quadratic relationship, a variant of a diode element is considered.

The paper describes the circuit of a universal function generator using quadratic approximations and capable of being implemented in the form of an attachment to a type EMU-8 analog computer.

Universal diode-function generators [1-5] based on the principle of piecewise-linear approximations are widely used in modern electronic analog computers for reproducing various nonlinear functions.

When it is necessary to reproduce a function with a comparatively large range of variation of the absolute magnitude of the first derivative, function generators of this type require the use of a large number of diode elements, which complicates the circuit and increases the error. In addition, discontinuities in the value of the first derivative of the reproduced function are introduced which, in many cases, is highly undesirable.

The disadvantages just cited can, to a large extent, be removed if nonlinear functions are approximated by segments of curves, in particular, by segments of parabolas [6-8].

However, the use of quadratic approximations entails a significant increase in the amount of computational work necessary to determine the partitioning along the argument axis, which has resulted in work being carried out to develop simplified analytic and graphical methods of calculating functions, and a variant of a universal function generator, based on the principle of quadratic approximations, has been developed.

Method of Piecewise-Quadratic Approximation of Nonlinear Functions

The problem of the piecewise-quadratic approximation of a nonlinear function f(x) amounts to the finding of a number of fixed argument values $x_1, x_2, x_3, \ldots$ which provides a second-order approximation polynomial with vertices $f(x_1), f(x_2), \ldots$

$$P_2(x) = a_0 + a_1 x + \sum_{i=1}^{n} b_i (x - x_i)^2,$$

where

$$b_i = \begin{cases} 0 & \text{for} \quad x \leqslant x_i, \\ B_i = \text{const} & \text{for} \quad x > x_i, \end{cases}$$

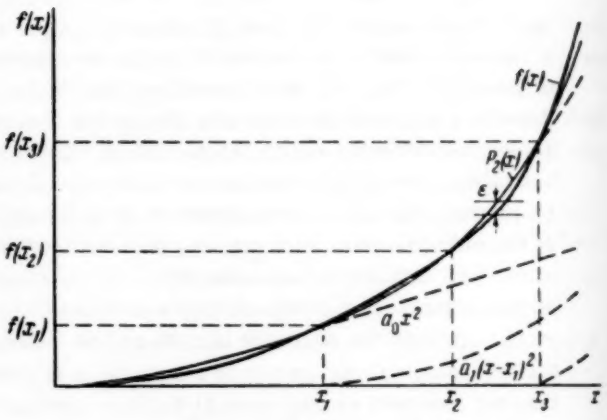

Fig. 1

to the curve f(x) to be reproduced such that, on the entire interval of argument variation, the absolute error of the approximation function does not exceed some previously given value ϵ (Fig. 1).

With respect to the function f(x), it is assumed that it is single-valued and continuous on the given interval of argument variation, and that it has a continuous second derivative everywhere on that interval with the possible exception of a finite number of first-order discontinuities.

For quadratic interpolation, the remainder term in Newton's interpolation formula has the form [9, 10]:

$$R(x) = \frac{f'''(\xi)}{3!}(x - x_1)(x - x_2)(x - x_3),$$

where $f'''(\xi)$ is the value of the reproduced function's third derivative at some point ξ inside the interval (x_1, x_3). The modulus of the error R(x) has its maximum value when $f'''(\xi)$ and the expression $|(x-x_1)(x-x_2)(x-x_3)|$ simultaneously attain their maxima:

$$|R(x)|_{\max} = \frac{|f'''(\xi)|_{\max}}{3!} |(x - x_1)(x - x_2)(x - x_3)|_{\max}.$$

We introduce the new variable $u = (x-x_1)/(x_3-x_1)$ and let $k = (x_2-x_1)/(x_3-x_1)$ and $h = x_3-x_1$.

Then,

$$|R(u, k)|_{\max} = \frac{|f'''(\xi)|_{\max}}{3!} |u(u-1)(u-k)|_{\max} h^3.$$

The function u(u-1)(u-k), in the region of variable variation $0 \le u \le 1$, $0 \le k \le 1$, attains its greatest value, equal to 4/27, at the points (u=1/3, k=1) and (u=2/3, k=0).

Thus, for this worst case, we have

$$R(u \quad k)_{\max} = \frac{4h^3 |f'''(\xi)|_{\max}}{27 \cdot 3!} = \frac{2h^3 |f'''(\xi)|_{\max}}{81}.$$

By assuming that $\max |R(u,k)| \le \epsilon$, we obtain the expression for determining the length of the partition segment $h = x_3 - x_1$ in the form

$$h \approx 3{,}43 \sqrt[3]{\frac{\epsilon}{|f'''(\xi)|_{\max}}}.$$

The worst case was taken in estimating the approximation error. In the general case, a more accurate consideration is very difficult. Experiment shows that, for a wide class of frequently arising functions, we may consider that

$$h = 4 \sqrt[3]{\frac{\epsilon}{|f'''(\xi)|_{\max}}}.$$

Example. It is required to reproduce the function $f(x) = x^3 \cdot 10^{-4}$ in the domain $0 \le x \le 100$. We find the law of argument partitioning when the approximation error is taken as $\epsilon = \pm 0.35$.

The third derivative of the given function equals the constant quantity $6 \cdot 10^{-4}$. Therefore, all the segments will be the same, and equal to

$$h_k = 4 \sqrt[3]{\frac{0{,}35 \cdot 10^4}{6}} = 33{,}6.$$

The total number of partition segments is three.

If the function f(x) is given graphically, then the threefold graphical differentiation necessary for the computation of the value of $f'''(\xi)$ is very difficult, and entails large errors. In this case, a graphical method should be used to determine the law of argument partitioning. This method is based on the replacement of a quadratic approximation to the given function f(x) with error ϵ by a piecewise-linear approximation to the first derivative f'(x) with error ϵ'. The problem reduces to the finding of the relationship $\epsilon' = \varphi(\epsilon, x)$. Figure 2 shows one segment of the quadratic second-order approximation polynomial $P_2(x)$. For $x = x_1$, $x = x_2$ and $x = x_3$, the function f(x) to be reproduced and the polynomial $P_2(x)$ have the common points:

$$f(x_1) = P_2(x_1),$$
$$f(x_2) = P_2(x_2), \quad f(x_3) = P(x_3).$$

The greatest error of approximation occurs when $x = x'$ and $x = x''$. It is obvious that for this

$$f'(x') = P_2'(x'), \quad f'(x'') = P_2'(x'').$$

In Fig. 3 there has been constructed the derivative $f'(x)$ of the given function, as well as the derivative $P'_2(x)$ of the approximating polynomial. The errors ϵ'_1, ϵ'_2, and ϵ'_3 occur respectively for $x = x_1$, $x = x_2$, and $x = x_3$.

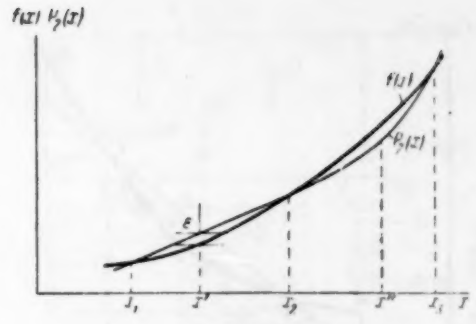

Fig. 2

From the approximation conditions, it follows moreover that

$$S_{ABC_1} = S_{CDE} = S_{DEF} = S_{FGK} = |\varepsilon|.$$

At the point $x = x'$, the angle between f'(x) and $P'_2(x)$ on the left, α_-, is not greater than the angle on the right, α_+: $\alpha_+ = \alpha_- + \Delta\alpha$, where $\Delta\alpha \ge 0$.

By replacing the curve $f_1(x)$ by straight lines on the segments AC and CE we find, from the equality of the areas $S_{\Delta ABC} = S_{\Delta CDE} \approx |\epsilon|$, that $\epsilon'_1 \le \epsilon'_2$.

Analogously, we can convince ourselves that $\epsilon'_3 \ge \epsilon'_2$.

We now consider F'(x) on the segment x ,x''. Here there is a one-sided linear approximation. Therefore, [11],

$$\epsilon'_2 \leqslant \frac{A_k (x'x'')^2}{8},$$

where A_k is the maximum value of the third derivative f'''(x) in the interval x'x''.

By assuming that point x_2 divides segment x'x'' in the ratio l, we find that

$$\epsilon'_1 \leqslant \frac{A_k (x'x'')^2}{8} l^2 \leqslant \epsilon'_2 l^2,$$

$$\epsilon'_2 \leqslant \frac{A_k (x'x'')^2}{8} (1-l)^2 \leqslant \epsilon'_1 (1-l)^2.$$

Thus,

$$\varepsilon = \frac{1}{2} S_{CFE} \approx \frac{1}{2} \left(S_{\Delta CEF} + S_{\Delta CER} + S_{\Delta EFO} \right) =$$
$$= \frac{x'x''}{2 \cdot 2} \left[\epsilon'_2 + l^2 \epsilon'_2 + (1-l)^2 \epsilon'_2 \right] =$$
$$= \frac{x'x''}{4} \epsilon'_2 (3l^2 - 3l + 2).$$

From whence

$$\epsilon'_2 = \frac{4\varepsilon}{x'x'' (3l^2 - 3l + 2)}.$$

The expression in the denominator assumes its minimum value for $l = \frac{1}{2}$. With this, $\epsilon'_2 \le 16\ \epsilon/5a$, where $a = x'x''$. This last formula gives the connection between the given error ϵ and the tolerance margin ϵ' and size of a.

In reproducing various functions, the lengths of neighboring partition intervals differ little from one another. Therefore, by measuring the quantity a on a constructed segment, we find from the last formula what the tolerance will be for the following segment. Inside the tolerance band ϵ', constructed with respect to f (x), we carry out a

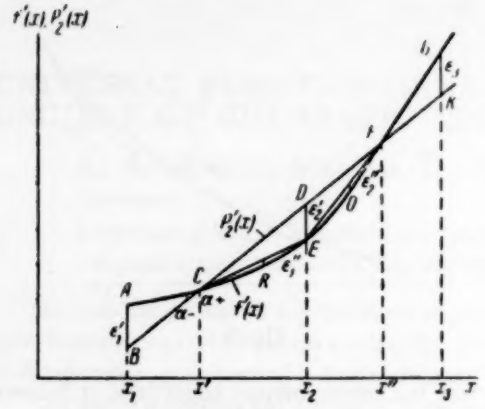

Fig. 3

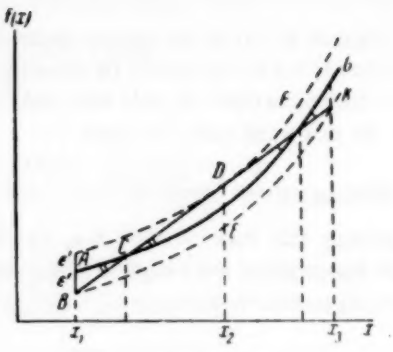

Fig. 4

two-sided linear approximation, as a result of which we find the size of the following partition interval.

It is easily shown that, with such a method of partitioning, the actual error will not be larger than the admissible approximation error ϵ. Indeed, let the admissible error ϵ' be computed from the previous segment. To determine the size of the partitioning segment, we construct the tolerance band with respect to f'(x) (Fig. 4). We then carry out a two-sided linear approximation.

Since ACB $\leq$ DCE, DFE $\leq$ GFK, then $S_{\Delta ABC} \leq S_{\Delta CDE} \leq \epsilon$ and $S_{\Delta FGK} \leq S_{\Delta FDE} \leq \epsilon$, i.e., with this we obtain a certain underestimate of the length of the partition interval, which leads only to a decrease in the error of approximation of the nonlinear function f(x) to be reproduced.

As the second derivative f''(x) increases, the curvature of the function f(x) also increases and, consequently, the lengths of the segments of the argument partition decrease. So that the magnitude of the error, defined by the previous segment, not exceed the tolerance on the given interval, the partitioning should be carried out from large segments to small ones, i.e., on the side of an increasing second derivative f''(x).

For the finding of the first segment, we may recommend a simplified method of successive approximations, shown on Fig. 5 for the function $f(x) = x^3 \cdot 10^{-4}$, where the graphical construction of the first segment is given. Letting the magnitude of $a_1 = 20$v, we construct the tolerance margin $\epsilon'_1 = 16\ \epsilon/5a_1 = 0.16\,\epsilon$, and we then carry out a piecewise linear approximation. We obtain the quantity $a_2 > a_1 = 20$. We construct the new tolerance margin $\epsilon'_2 = 16\ \epsilon/5a_2$, and carry out a piecewise linear approximation which determines a_3. It is obvious that the optimal length of the segment will be $a_1 \leq a_3 \leq a \leq a_2$. As the interval length varies within these limits, the maximum actual error ϵ'_i decreases from ϵ'_1 to ϵ'_2, varying along a certain curve AB which is close to being a straight line. At the same time, the admissible error varies along a hyperbola which passes through points C and D, which correspond to the maximum values of ϵ'_i for $a_1 = 20$ and $a_2 = 24$. If we now replace the hyperbola by the straight line CD we find, on Fig. 5., the point 0 of intersection which, with an accuracy sufficient for a graphical method, gives the magnitude of the error ϵ' for the first and second intervals of the argument partitioning.

Circuit of the Quadratic Diode Element

For obtaining segments with quadratic dependencies in the present work, we used the diode element represented on Fig. 6, a [8]. To the diode are applied input vol-

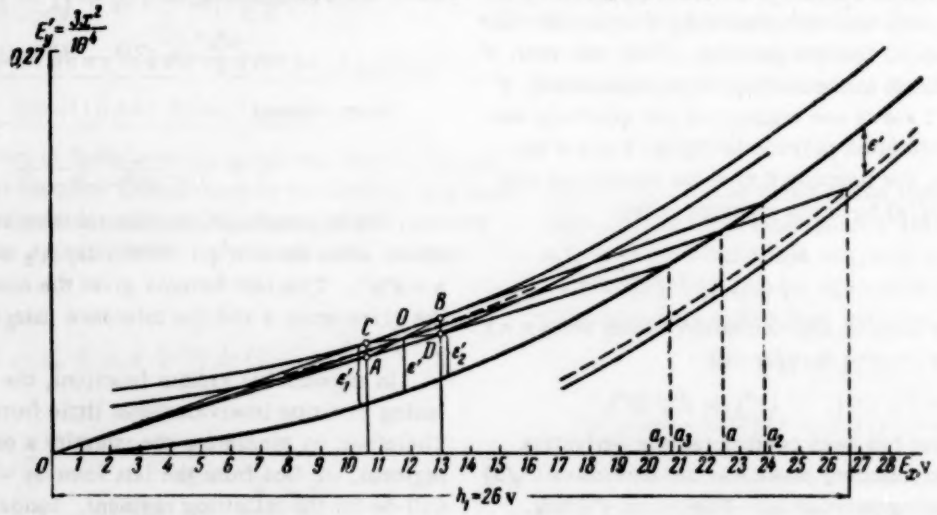

Fig. 5

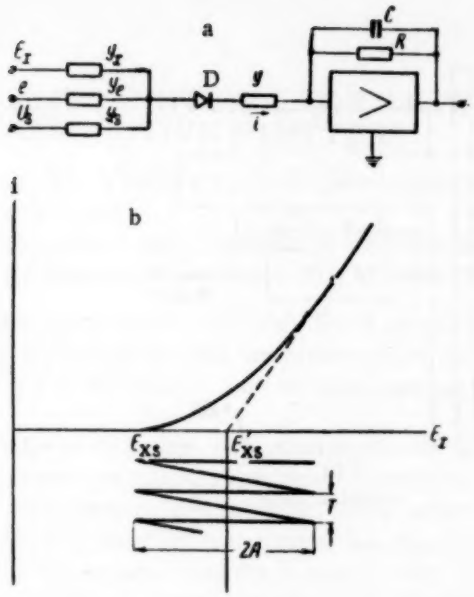

Fig.6

tage E_x, reference voltage e and high-frequency sawtooth voltage U_s. The diode element's transconductance characteristic is determined by the conductance Y.

If the high-frequency voltage U_s is not applied to the diode, the dependence of the current i on the input quantity E_x has the form shown on Fig. 6,b by the dotted line. For $E_x = E'_{xs} = eY_e/Y_x$, diode D is opened, and thereafter the current i increases proportionately to input voltage E_x.

If a sawtooth voltage of amplitude A is applied to the diode via Y_s, the diode will open for lower values of the input voltage:

$$E_x = E_{xs} = e\frac{Y_e}{Y_x} - A\frac{Y_s}{Y_x}.$$

By using the principle of superposition, we find the average value of the diode element's current:

$$i_{av} = \begin{cases} 0 & \text{for } E_x \leqslant E_{xs}, \\ \dfrac{Y}{YAY_0Y_s}(Y_xE_x - Y_ee + Y_s A)^2 & \text{for } E_x > E_{xs}. \end{cases}$$

Here,

$$E_{xs} = e\frac{Y_e}{Y_x} - A\frac{Y_s}{Y_x},$$

$$Y_0 = Y_x + Y_e + Y_s + Y.$$

For getting a sawtooth voltage, a generator circuit, shown on Fig. 7, was developed. A blocking generator was executed based on triode V_1, while pentode V_3 was used as a discharge tube. Charging condenser C_1 is connected in the anode circuit of V_3. During the blocking process, the condenser is charged up to the potential U_{max} close to the anode voltage. Tube V_1 is cut off and condenser C_1 discharges through tube V_3. Because of the current-stabilizing action of the pentode, the discharge of C_1 occurs linearly. As soon as the voltage becomes equal to $U_{C_1} = U_{unb.V_1}$, tube V_1 opens and the blocking process

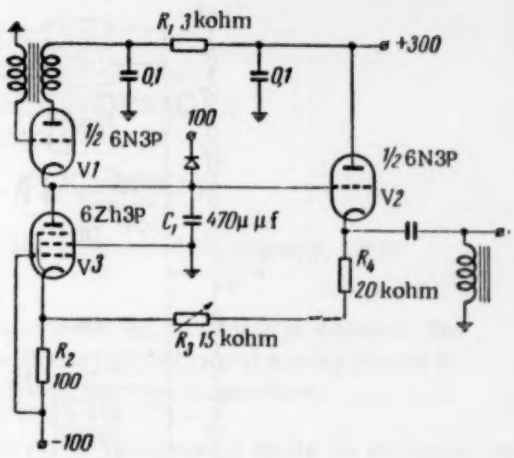

Fig. 7

repeats. The generated voltage is applied to the grid of cathode follower V_2. Part of the output voltage is tapped off by the voltage divider formed by resistors R_2, R_3, and R_4, and is applied to the cathode of V_3. The magnitude of the positive feedback can be controlled by resistor R_3, thus varying, within very wide limits, the linearity of the output voltage.

To stabilize the sawtooth amplitude, we use a reference voltage of +100v, limiting the growth of potential on capacitor C_1. The frequency of the generated voltage is determined by the capacitance of condenser C_1 and by the magnitude of the anode current of pentode V_3.

With the nominal values shown on Fig. 7, the circuit provides a generated sawtooth voltage of frequency 25 kcps and output voltage amplitude of 90v. With line-voltage variations of ± 10%, the sawtooth amplitude varies by ± 5%.

Complete Function-Generator Circuit and Its Technical Characteristics

Vacuum tube diodes type 6Kh2P were used as the function generator's diode elements. The transition to vacuum tubes resulted from the great difficulties which arose when germanium diodes were used, due to their low back impedance and their instability under various temperature conditions.

The functional schematic of the universal diode-function generator is given in Fig. 8. In it are included two operational amplifiers A_1 and A_2. Applied to the diode elements DE are a high-frequency sawtooth voltage and a reference voltage from a low-ohmic divider. The low-ohmic divider is executed in the form of individual sections, each of which consists of six wire resistances: r_1 = 60 ohm, r_2 = 120 ohm, r_3 = 240 ohm, r_4 = 420 ohm, r_5 = 840 ohm and r_6 = 1680 ohm. For setting up the law of argument partitioning, part of these resistances are short-circuited by means of a twelve-position switch with inserts in its panel. The total impedance of the low-ohmic dividers is always so chosen that a current of 16.7 ma flows through them. With this, the increment of the argument between the switching points of any neighboring diode elements is con-

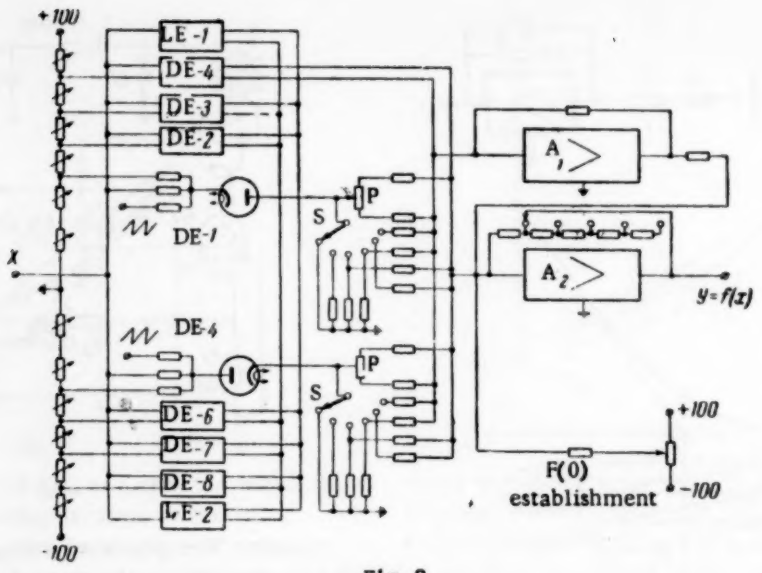

Fig. 8

trolled in the limits of 0 to 56v, varying by discrete steps of 1 volt.

In the function generator, there are eight quadratic diode elements, for which the switching voltage can be controlled, and two linear elements LE, made up of passive-resistor circuits, with switching voltages by zero. By applying the voltage after the diodes, either directly to A_2 or through sign-inverter A_1, we can obtain increments of the function's slopes in all four quadrants. The magnitude of these increments can be controlled discretely and smoothly by, respectively, switch S and potentiometer P (Fig. 8).

The function generator's complete circuit also contains an assembly for establishing the function's initial value.

SUMMARY

1. The function generator constructed in accordance with the principle of quadratic approximations has the following basic advantages in comparison with function generators, in which piecewise-linear approximations are used.

a) A significant reduction in the number of segments of the argument partitioning which are necessary to reproduce the function with the same error. This permits one to simplify the universal function-generator's circuit, to decrease markedly the number of vacuum tubes, to ease significantly the set-up of the functions and the adjustments of the generator.

b) The function generator provides smooth reproduction of functions, and eliminates discontinuities in the first derivative.

2. With this, the universal function generator constructed on the basis of the principle of quadratic approximation possesses the following disadvantages.

a) The device's pass band is determined by the sawtooth generator's frequency which, due to technical difficulties, can not be made higher than 30 to 50 kcps. Therefore, the function generator cannot have a larger band than 1 kcps.

b) The sawtooth voltage has a high-frequency component, so that the mounting block requires careful screening.

Despite these disadvantages, the circuit developed for the universal function generator can find practical employment in electronic models and in various computing devices.

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RELIABILITY CRITERIA FOR AUTOMATIC RELAY DEVICES WITH RADIOACTIVE EMITTERS

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Criteria are given for the reliability of automatic relay devices with sources of radioactive emission. The relationship is given connecting the parameters of the relay devices with the probability that a relay remain in a given state and, in addition, with the mean number of "false" transfers of the relay in unit time.

Calculated relationships were given in [1] which allow one to choose the minimum necessary activity of emitters in relay devices from the given limits on the admissible number of standard deviations of the operating utilization of the detector from the threshold level. In the work just cited, one limited oneself, in dealing with the statistical characteristics of the process, to an approximate representation of the potential distribution at the integrator relay's output as a Poisson distribution, or as a Gaussian distribution close to a Poisson one. The recommendations based on these considerations turn out to be of practical utility in determining the parameters of ordinary industrial relay devices. However, in many relay devices in which high reliability or speed of action are required, the accuracy of the computations based on the methodology of [1] may turn out to be inadequate.

Moreover, in the methodology given, the reliability of a device's operation was given only by the probability of a relay's remaining in the proper states. For many devices, in addition to this criterion, it is of great practical value to use, as a reliability criterion, the mean number of "false" transfers to be expected in unit time.

In the present work we render more exact the limits of applicability of the first approximation of [1] for various values of relay parameters. Methods are given for the numerical calculation of reliability to any desired degree of accuracy. Reliability is defined in terms of the probability of "false" relay transfers. Methods are given for the computation of the mean number of "false" transfers in unit time to any desired degree of accuracy. Graph are given for the determination of reliability from the probability of remaining in a "false" state, and the number of "false" transfers for various combinations of the parameters of relay devices.

As was shown in [2] and [3] the characteristic distribution function of the random variable

$$x = \frac{V}{V_0} \tag{1}$$

for a Poisson distribution of the frequency of the applied signal n(t) and reaction function $\Phi(t)$ of the device has the form:

$$\vartheta(\lambda) = \exp\left\{\int_{-\infty}^{T}\left[e^{i\lambda\Phi(T-t)}-1\right]n(t)\,dt\right\}. \tag{2}$$

From (2) we immediately obtain the expression for the cumulants of the distribution

$$k_\mu = \int_{-\infty}^{T}\Phi^\mu(T-t)\,n(t)\,dt. \tag{3}$$

We shall consider devices in which an RC network is used an the integrator. Then, the reaction function will have the form:

$$\Phi(t) = e^{-\frac{t}{RC}}. \tag{4}$$

In (1), V is the potential at the RC network's output and V_0 is the increment of potential at the RC network's output at the moment when a pulse is applied. In the steady state, V_0 is a constant. During transient responses, V_0 remains practically constant if the pulse amplitude is very much greater than the size of V_0 over its entire range of variation during the transient response. This condition holds with sufficient accuracy in all relay devices which operate as velocity measurers with linear scales.

A direct use of (2) for calculating a probability density and a distribution function would lead to extremely cumbersome computations. We shall consider two series expansions for the desired probability density p(x) and the distribution function F(x), one of which is useful for large values of

$$\nu = nRC, \tag{5}$$

and the other for small values of ν.

It was shown, in [2, 3] that, as $\nu\to\infty$, the desired distribution tends to a normal one. For computing the correction to the normal distribution, Rice suggested the use of a series first introduced by Edgeworth (an expansion in terms of the derivatives of the normalized density of a normal distribution). For the probability density, this series has the form:

$$p(x) = \frac{1}{k_2^{1/2}}\left[\varphi\left(\frac{x-k_1}{k_2^{1/2}}\right) - \frac{1}{3!}\frac{k_3}{k_2^{3/2}}\varphi^{(3)}\left(\frac{x-k_1}{k_2^{1/2}}\right) + \right.$$
$$+ \frac{1}{4!}\frac{k_4}{k_2^2}\varphi^{(4)}\left(\frac{x-k_1}{k_2^{1/2}}\right) + \tag{6}$$
$$+\frac{10}{6!}\frac{k_3^2}{k_2^3}\varphi^{(6)}\left(\frac{x-k_1}{k_2^{1/2}}\right) - \frac{1}{5!}\frac{k_5}{k_2^{5/2}}\varphi^{(5)}\left(\frac{x-k_1}{k_2^{1/2}}\right) -$$
$$\left. - \frac{35}{7!}\frac{k_3k_4}{k_2^{7/2}}\varphi^{(7)}\left(\frac{x-k_1}{k_2^{1/2}}\right) - \frac{280}{9!}\frac{k_3^3}{k_2^{9/2}}\varphi^{(9)}\left(\frac{x-k_1}{k_2^{1/2}}\right) + \ldots\right],$$

and, for the distribution function,

$$F(x) = 0{,}5 + \Phi\left(\frac{x-k_1}{k_2^{1/2}}\right) - \frac{1}{3!}\frac{k_3}{k_2^{3/2}}\varphi^{(2)}\left(\frac{x-k_1}{k_2^{1/2}}\right) + \\ + \frac{1}{4!}\frac{k_4}{k_2^2}\varphi^{(3)}\left(\frac{x-k_1}{k_2^{1/2}}\right) + \\ + \frac{10}{6!}\frac{k_3^2}{k_2^3}\varphi^{(5)}\left(\frac{x-k_1}{k_2^{1/2}}\right) - \frac{1}{5!}\frac{k_5}{k_2^{5/2}}\varphi^{(4)}\left(\frac{x-k_1}{k_2^{1/2}}\right) - \\ - \frac{35}{7!}\frac{k_3k_4}{k_2^{7/2}}\varphi^{(6)}\left(\frac{x-k_1}{k_2^{1/2}}\right) - \frac{280}{9!}\frac{k_3^3}{k_2^{9/2}}\varphi^{(8)}\left(\frac{x-k_1}{k_2^{1/2}}\right) + \ldots \quad (7)$$

In using the Edgeworth series, the greatest difficulty is in computing the distribution cumulants in the case when n is not constant. If the velocity of counting does not depend on time, then

$$k_\mu = \int_{-\infty}^{T} e^{\frac{\mu(t-T)}{RC}} n(t)\,dt = \frac{nRC}{\mu} = \frac{\nu}{\mu} \quad (8)$$

and the solution is essentially simplified.

We shall consider such relay devices for which the relative fraction of the time spent in a stationary state is quite large. Then, any consideration of the reliability of the relay's operation can be reduced to a consideration of its reliability in the stationary states.* For this case, (6) and (7) are written in the form

$$p(x) = \frac{0{,}5}{\sqrt{\nu}}\Phi_1\left(\frac{z}{\sqrt{2}}\right) - \frac{0{,}0278}{\nu}\Phi_4\left(\frac{z}{\sqrt{2}}\right) + \\ + \frac{0{,}00520}{\nu^{3/2}}\Phi_5\left(\frac{z}{\sqrt{2}}\right) + \frac{0{,}00617}{\nu^{3/2}}\Phi_7\left(\frac{z}{\sqrt{2}}\right) - \\ - \frac{0{,}000834}{\nu^2}\Phi_6\left(\frac{z}{\sqrt{2}}\right) - \frac{9{,}87\cdot10^{-5}}{\nu^2}\Phi_8\left(\frac{z}{\sqrt{2}}\right) - \\ - \frac{3{,}58\cdot10^{-6}}{\nu^2}\Phi_{10}\left(\frac{z}{\sqrt{2}}\right) + \ldots, \quad (9)$$

$$F(x) = 0{,}5 + \Phi(z) - \frac{0{,}0278}{\sqrt{\nu}}\Phi_3\left(\frac{z}{\sqrt{2}}\right) + \\ + \frac{0{,}00522}{\nu}\Phi_4\left(\frac{z}{\sqrt{2}}\right) + \frac{0{,}000773}{\nu}\Phi_6\left(\frac{z}{\sqrt{2}}\right) - \\ - \frac{0{,}000832}{\nu^{3/2}}\Phi_5\left(\frac{z}{\sqrt{2}}\right) - \frac{1{,}15\cdot10^{-4}}{\nu^{3/2}}\Phi_7\left(\frac{z}{\sqrt{2}}\right) - \\ - \frac{4{,}20\cdot10^{-6}}{\nu^{3/2}}\Phi_9\left(\frac{z}{\sqrt{2}}\right) + \ldots, \quad (10)$$

where

$$\Phi(z) = \frac{1}{\sqrt{2\pi}}\int_0^z e^{-\frac{V^2}{2}}dV \quad (11)$$

is a normalized Laplace function, $\Phi_n(z/\sqrt{2})$ is a derivative of the error probability integral and

$$z = \frac{x-k_1}{k_2^{1/2}} \quad (12)$$

is the multiplicity of the standard deviation.

It is easily shown [1,4] that

$$x = n_0RC, \quad (13)$$

$$k_1 = nRC, \quad (14)$$

$$k_2^{1/2} = \sqrt{\frac{nRC}{2}}. \quad (15)$$

After substituting (13)-(15) in (12), we get

$$z = \frac{2RC(n_0-n)}{\sqrt{2RCn}}, \quad (16)$$

where n_0 is the relay's operation threshold, i.e., the counting speed for which both relay states are equally probably. It is obvious that for the states in the absence of a controlled object on the control line (state 1) the magnitude of n will exceed n_0, and these states will be characterized by negative values of z. For the other state (state 2), the quantity z will be positive.

The essential disadvantage of (9) amd (10) is their nonuniform convergence. These series obviously have poor convergence for large values of z and small values of ν. For $\nu = 0$, the distribution function degenerates to the Heaviside unit function $\eta(x)$:

$$\eta_1(x) = \begin{cases} 0 & \text{for } x < 0, \\ 1 & \text{for } x \geqslant 0. \end{cases} \quad (17)$$

Therefore, for large z and small ν, it is meaningful to use expansions based on the function $\eta(x)$.

One is led to this type of expansion by a method analogous to that used by Maslov and Povzner [5].

As one may easily convince oneself, the desired probability density p(x) is the solution of the equation

$$\frac{\partial u}{\partial y} = \int_0^1 \frac{u(x-x',\ y) - u(x,\ y)}{x'}\nu\left(\ln x' + \frac{T}{RC}\right)dx' \quad (18)$$

with initial conditions

$$u(x,\ 0) = \eta_1(x) \quad (19)$$

and for the parameter value y = 1.

For y = 1, the distribution function F(x) also coincides with the solution of (18), but with the initial conditions

$$u(x,\ 0) = \eta(x). \quad (20)$$

With initial conditions (20), the solution of (18) can be obtained either by the methods of finite differences, or in the form of a series

$$F(x) = e^A[\eta(x)] = \eta(x) + A[\eta(x)] + \\ + \frac{1}{2!}A^2[\eta(x)] + \ldots + \frac{1}{m!}A^m[\eta(x)], \quad (21)$$

where the letter A denotes the integral operation

$$A[u(x)] = \int_0^1 \frac{u(x-x') - u(x)}{x'}\nu\left(\ln x' + \frac{T}{RC}\right)dx'. \quad (22)$$

The convergence of (21) is also nonuniform. For small x, the series converges rapidly, and it is meaningful to use it. For large x, it is more convenient to use finite-differences methods.

* A criterion for reliability of relay operation during the transient response could be the probability of just one transfer during the response time.

Computing the distribution function by some method or other, we can find the average time that the relay remains in a given state.

In addition, as has been stated, it is necessary to compute the mean number of "false" transfers per unit time. For an inertialess relay, this problem leads to the determination of the number of intersections of the current potential value with the potential value V_n for which the relay changes state. For state 1, $|\nabla| > |V_n|$, and the problem reduces to finding the number of intersections coming down from above ($S\downarrow$). For state 2, $|\nabla| < |V_n|$, it is necessary to compute the number of intersections going up from below ($S\uparrow$).

The general formula, well known in the literature [6], for finding the stated quantity ("mean number of blips") requires, for its practical use, a tremendous amount of computation.

In our case, finding the expression for the mean number of intersections poses no difficulty.

We consider a small interval of dimensionless time $\tau = t/RC$, $(\tau, \tau + \Delta\tau)$. The average number of intersections of some level x_0, both coming down from above and going up from below, in this interval is an infinitesimal quantity of the order of $\Delta\tau$. As is known [6], this quantity can be replaced, with an accuracy of second-order infinitesimals in $\Delta\tau$, by the probability $P_{\tau,\Delta\tau}$ of one intersection going up from below and the probability $P_{\tau,\Delta\tau}$ of one intersection coming down from above. Our aim is to find the following limits:

$$S^{\uparrow} = \lim_{\Delta\tau\to 0} \frac{1}{\Delta\tau} P^{\uparrow}_{\tau,\Delta\tau}, \qquad S^{\downarrow} = \lim_{\Delta\tau\to 0} \frac{1}{\Delta\tau} P^{\downarrow}_{\tau,\Delta\tau}. \tag{23}$$

Thus, the quantities $P^{\uparrow}_{\tau,\Delta\tau}$ and $P^{\downarrow}_{\tau,\Delta\tau}$ can be found with an accuracy of second-order infinitesimals in $\Delta\tau$.

Each of the realizations of the process under consideration increases by a unit jump when a pulse is applied, and decays exponentially in the intervals between pulses.

A realization of the process can intersect the level x_0 from above to below if, and only if, the quantity $x(\tau)$ at time τ lies between the limits x_0 and $x_0 e^{\Delta\tau}$ and, in the time interval $\tau, \tau + \ln(x/x_0)$, no pulse is applied. Thus,

$$P^{\downarrow}_{\tau,\Delta\tau} = \int_{x_0}^{x_0 e^{\Delta\tau}} p(x)\left(1 - \nu(\tau)\ln\frac{x}{x_0}\right)dx \approx x_0 p(x_0)\,\Delta\tau, \tag{24}$$

where the last equality holds with the accuracy required.

It immediately follows from (24) that

$$S^{\downarrow} = x_0 p(x_0). \tag{25}$$

If we now make the transition from dimensionless time to ordinary time, and take (1), (9), and (12)-(16) into account, we obtain the definitive formula

$$S^{\downarrow} = n_0 p(x). \tag{26}$$

Intersection of level x_0 from below upwards is possible if, and only if, $x(\tau)$ lies between $x_0 - 1$ and x_0 and a pulse is applied during an interval less than $\ln[x/(x-1)]$. This means that

$$P^{\uparrow}_{\tau,\Delta\tau} = \nu(\tau)\,\Delta\tau \int_{(x_0-1)e^{\Delta\tau}}^{x_0} p(x)\,dx + \nu(\tau) \times \times \int_{x_0-1}^{(x_0-1)e^{\Delta\tau}} p(x)\ln\frac{x}{x_0-1}\,dx \approx \approx \nu(\tau)\,\Delta\tau \int_{x_0-1}^{x_0} p(x)\,dx. \tag{27}$$

Whence,

$$S^{\uparrow} = \nu(\tau)\int_{x_0-1}^{x_0} p(x)\,dx = \nu(\tau)\,[F(x_0) - F(x_0-1)]. \tag{28}$$

For values of $x_0 \gg 1$ (this corresponds, as is easily seen, to $n_0 RC \gg 1$), we can set

$$S^{\uparrow} = \nu(\tau)\,p(x_0). \tag{29}$$

After simple transformations, we finally obtain†

$$S^{\uparrow} = np(x). \tag{30}$$

Starting from what has been presented, we can write the expression for the average blip duration. For state 1 we get

$$T^{\downarrow} = \frac{F(x)}{n_0 p(x)}, \tag{31}$$

and for state 2

$$T^{\uparrow} = \frac{1 - F(x)}{np(x)}. \tag{32}$$

It is of interest to find the expressions for the distribution of durations in "false" states and for the distribution of the number of "false" intersections. This is a complicated problem, and will not be treated in the present work. At the same time, for example, it is easily seen that, with the condition

$$\frac{1}{S^{\downarrow}} > T^{\downarrow} \quad \text{or} \quad \frac{1}{S^{\uparrow}} > T^{\uparrow} \tag{33}$$

the distribution of the number of "false" intersections is close to a Poisson distribution.

We now consider concrete results of computations based on the method presented here.

There is great interest in determining the quantities $1 - F(x)$, $F(x)$, $S^{\downarrow}$, $S^{\uparrow}$, $T^{\downarrow}$, $T^{\uparrow}$ for a wide interval of values of ν and $\underline{z}$.

Of practical interest are the solutions for large values of $\underline{z}$, corresponding to automation and control systems in which "false" transfers lead to very undesirable consequences, and for small ν. Such solutions are obtained effectively by means of computers, using the method presented above [(17)-(22)].

† For reliability characteristics based on the number of "false" intersections, one can also use functions of S which take on values from 0 to 1, for example, P(S) = 1/(1+S).

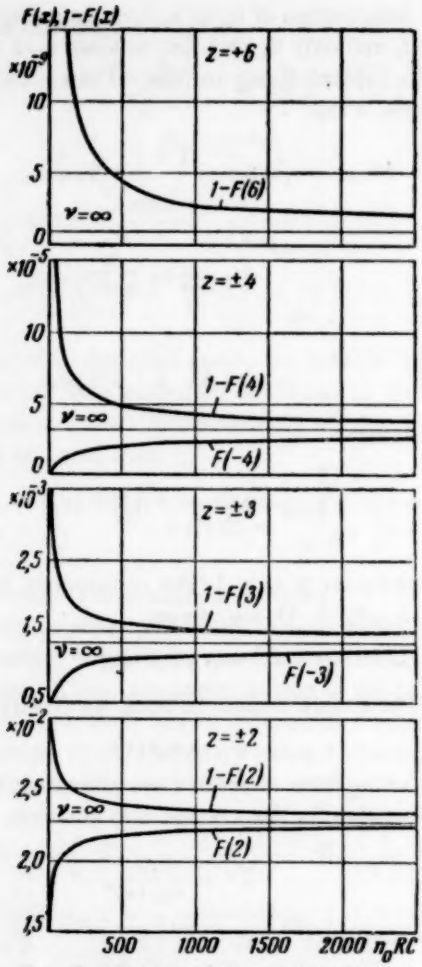

Fig. 1. Values of the probability of the potential remaining in a "false" state.

For widely used relay devices working with low values of z and large values of ν, one can use the Edgeworth series (9) and (10) for the direct computation of the quantities listed above. The values of $\Phi_n(z/\sqrt{2})$ are computed by means of Hermite polynomials:

$$\Phi_n\left(\frac{z}{\sqrt{2}}\right) = (-1)^{n-1} 2^{\frac{n-1}{2}} \frac{2}{\sqrt{\pi}} e^{-\frac{z^2}{2}} H_{n-1}(z). \quad (34)$$

Figure 1 shows the values of the quantities F(x) and 1-F(x) for several values of z and $n_0RC \ll 2500$. These quantities equal the mean relative time the potential remains in a "false" state for, respectively, state 1 and state 2. The computations were made with all the terms actually written above in (10). For convenience in comparing the results with those previously obtained [1], the product n_0RC has been laid out along the axis of abscissas, being defined, as follows from (5) and (16), by the expression

$$n_0RC = \nu + 2\sqrt{\frac{\nu}{2}}. \quad (35)$$

The first two terms in (10) do not depend on ν, and give a solution in a Gaussian approximation ($\nu = \infty$). As is clear from Fig. 1, in determining the relative time spent in a "false" state, the deviation from the Gaussian approximation is the larger, the larger is z and the smaller is the product n_0RC.

It should be mentioned that, for state 1, the computations carried out in the Gaussian approximation lead to an increase in the actual reliability of state 1 in comparison with the given value. With this, the value of the minimum necessary activity of the radiation source of 1 is somewhat overstated. In connection with this, as is clear from Fig. 1, the reliability of state 2 will be lower than that which follows from the computations in the Gaussian approximation, from whence the value of the limiting admissible activity of the radiation source turns out to be somewhat overstated. However, in devices in which the controlled object provides a significant attenuation of the emission, the magnitude of the limiting admissible activity significantly exceeds the value of the minimum necessary activity, as is clear from the calculations made in [1]. For these devices, one need not take into account, in calculating source activity, the deviations from the Gaussian approximation for state 2.

The deviations changing, by an order of magnitude, $0.5-\Phi(z)$ occur at $z = \pm 2$ for $n_0RC \lesssim 10$, at $z = \pm 3$ for $n_0RC \lesssim 30$ and at $z = \pm 4$ for $n_0RC \lesssim 50$. Therefore, the Gaussian approximation, as was already noted [1], encompasses a wide circle of problems, but is inadequate when one deals with devices designed for high reliability (large z), high speed of response (low RC) in conjunction with high sensitivity of the relay to radiation (small n_0).

At $z = -6$, the magnitude of $F(-6)$ is $(0.2 \text{ to } 0.5)10^{-9}$ for values of n_0RC from 500 to 2500. In calculating the value of $F(-6)$ for $n_0RC < 500$, the number of terms written in (10) turned out to be inadequate.

The table gives the upper limits on the error of computing the quantities 1-F(x) and f(x), using all the terms actually written in (10). The errors are stated to within 1%. As is clear from the table, the error in computing the relative time in a "false" state falls with decreasing ν and with increasing z.

The accuracy of the determination of the number of "false" transfers is determined by the accuracy of determining the quantity p(x), as is clear from (26) and (30). The accuracy of computing the quantities $S\downarrow$ and $S\uparrow$ varies in approximately the same manner as that for the relative duration of the relay in a "false" state (Cf. the table).

For various values of z and RC, Fig. 2 shows the number of "false" intersections of potential per unit time as a function of n_0. At $z = 6$, the number of terms of (9) taken into account permit the computations to be made only for relatively large values of n_0RC. For example, the computations give $S\uparrow \approx \text{const} \approx 2\cdot10^{-9}$ for $z = 6$, $RC = 50$ and $n_0 < 200$.

As is clear from Fig. 2, the number of intersections $S\downarrow$ and $S\uparrow$ do not equal each other for the same multiplicity of the standard deviation. This is particularly apparent for large values of z.

TABLE Error in Computing the Relative Time the Potential Remains in a "False" State

n_0	Error less than δ, %							
	z							
	+2	−2	+3	−3	+4	−4	+6	−6
10	0	1	2	90	11	448		
20	0	0	1	8	6	325		
50	0	0	0	2	3	47		
100	0	0	0	0	1	9	28	
200	0	0	0	0	1	2	18	
500	0	0	0	0	0	1	8	148
1000	0	0	0	0	0	0	4	32
2500	0	0	0	0	0	0	2	6
5000	0	0	0	0	0	0	1	2

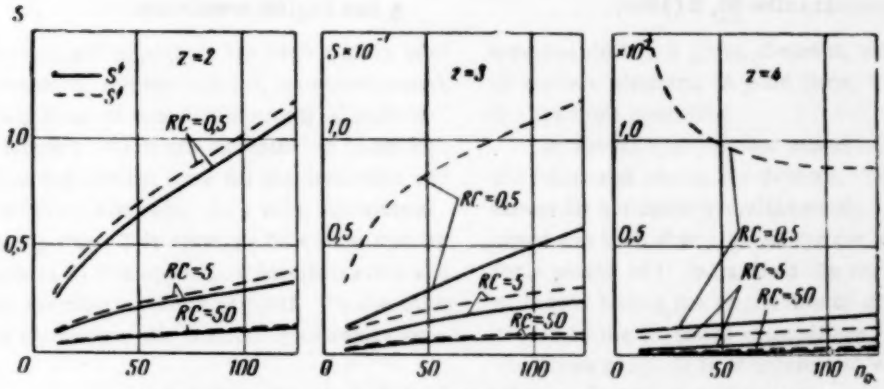

Fig. 2. Average number of "false" intersections of the potential per unit time.

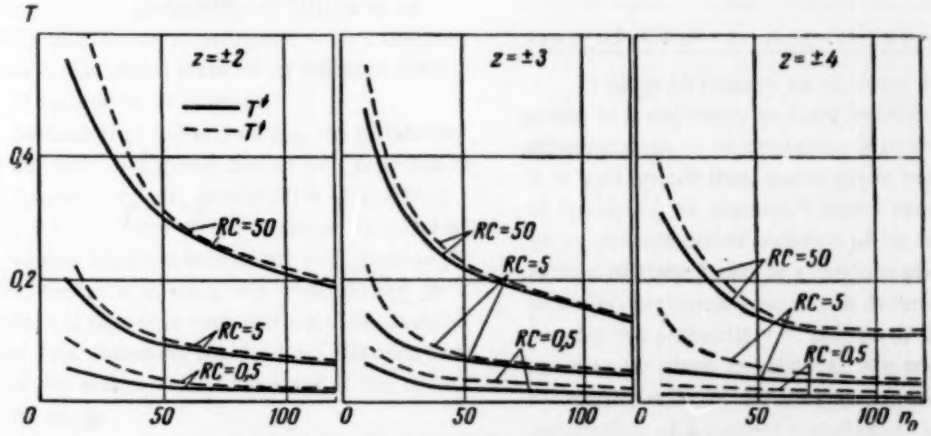

Fig. 3. Average time of "false" intersections.

Figure 3 shows the average time the potential remains in a "false" state for various values of z, n_0 and RC, calculated by (31) and (32).

It should be mentioned that the inertia of the electrical (for example, lamps or bells in the anode circuit of power amplifier relays) or electromechanical relay output (electromechanical relay) can lead to a certain lack of coincidence of the relative time the relay contacts (lamps, bells) remain in a "false" state as compared with the values computed by the method presented above.

On the one hand, a sufficiently inertial output at relay operation will not react on intersections less than some value T_0, determined by this inertia. On the other hand, relays with long release times will lead to a drawing out of the time that the relay device remains in a "false" state after the "false" intersection of the potential has terminated.

With respect to "false" relay transfers, it may be remarked that their number can be somewhat less than the number of "false" intersections of potential, due both to intersections lasting less than T_0 and to the appearance of more than one intersection while the contacts are to be found in a "false" state. These circumstances act to increase the relay's reliability, defined in terms of number of intersections, but limit its speed of response to a well-known extent.

Based on the results given above, one can calculate the relative duration of relay contacts in "false" states and the number of "false" contact transfers for concrete relays with known characteristics. Since the number of known relay types is very large, this question requires special consideration.

In conclusion, the authors wish to thank L. K. Tatochenko for the great interest shown in the present work, and for a number of very useful comments.

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ESTIMATES, BASED ON USAGE, OF THE SERVICE LIFE OF CONTACT APPARATUS

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The region of useful life of contact apparatus is determined as a function of its modes of operation and of the given admissible number of transfers. Starting from the service life, we estimate the increment of use expenditure per element.

Hitherto, relay contact apparatus has been widely used in systems for automation, remote control, communication, etc. The breakdowns (out-of-order conditions) of individual elements and blocks† which are unavoidable under conditions of use require significant time for the detection and replacement of the faulty element. As a rule, the system must stand idle during this. The system's idle time can be reduced to a minimum by the opportune determination and replacement of the element or block at fault. On the other hand, this measure eliminates the possibility of faulty system operation.

It is well known that the probability of failure of the electromechanical (receiving) portion of a relay element is very much less than the probability of failure of its contacts [1]. This allows one to estimate, with a definite degree of accuracy, the useful state of an element from the probability of failure of its contacts.

A general methodology for estimating the reliability of operation of contacts in a given circuit as a function of such factors as contact pressure, possibilities of sparking or arcing, changes in the form of the contacts' surfaces and other physical reasons for their breakdown are given in [1, 2]. The essential obstacle to using this methodology in usage computations is that it is very difficult, in practice, to determine the data necessary for the computations and, consequently, for the estimation of the state of the contracts during their usage.

The method, described by N. Taylor [3] (the marginal checking method), for inspecting allows one to detect elements which are on the point of breaking down and to replace them. However, this method has been developed as applied to electronic apparatus. Its practical use for the estimation of contact apparatus is not worthwhile, since it entails superfluous operations.

The common feature of these methods is that testing and verifying of element states must be carried out directly at the location where the elements are mounted or by removing them which, in many cases, can turn out to be impossible or inadmissible in practice.

It follows from this that it would be of interest to develop a method for the indirect estimate of the state of contact apparatus. Such a method, which would permit one to determine, if only approximately, the length of serviceable life a given element, would make it possible to replace elements in good time, without detriment to the system's operation.

In actual systems, the modes of operation of different elements and blocks are diverse. Therefore, their breakdowns do not occur simultaneously. If the conditions imposed are such that a given system must operate properly for a period of H years, then the replacement of elements or blocks before the expiration of this period entails an increase in the material expenditure, a "use" expenditure in the given case. It is of interest to estimate the variation of these latter expenditures.

Admissible Duration of Continuous Operation of Contact Apparatus

In order to replace an element at the breakdown point, it is necessary to know beforehand what the admissible duration of its operation is in the given mode of work. It is well known that, under given conditions and modes of operation, an element's useful service life is determined by the number of transfers of its contacts [1]. Since failure of these latter is a random phenomenon, one of the principal problems amounts to determining the distribution law for the probability of failure of the contacts of an element under given conditions. We note that certain authors, for example, V. A. Moroz [4], in their works start with the assumption of a normal distribution law.

In this work we shall adopt the convention that, on the basis of tests and statistical data on the operation of elements whose mode of operation is given by the parameters and structure of the scheme under consideration, we shall have determined the distribution law of the probability of contact failure, and that from this we shall have found the admissible number of transfers of the latter, N_{adm}.

If we start from this condition, then we can determine the admissible duration of continuous operation

† By the term "block" in the present paper, we understand a set of elements implementing a definite function and which are found under identical operating conditions.

of an element for a given speed of transfer of its contacts during a call cycle. For electromagnetic elements, this is

$$T_{adm} = \frac{N_{adm}}{v \cdot 3600} \text{ hr}, \quad (1)$$

where v is the speed of contact transfers, in units of sec^{-1}.

Definite types of reed frequency elements, in the process of signal reception, transfer the contacts in each period of current (voltage) oscillation, coinciding with their natural frequency (Cf., for example, [5]). If oscillations of frequency f, to which the frequency element is tuned, exist during time t, then the total number of contact transfers during this time will be ft. Consequently,

$$T_{adm} = \frac{N_{adm}\tau}{3600\, ft} \text{ hr}, \quad (2)$$

where τ is the period of a call cycle, cycle, in seconds ‡.

Thus, the admissible duration of continuous operation of a reed element depends both on its natural tuned frequency, and on the time during which the signal exists.

Domain of Proper Operation of Contact Apparatus

If a given circuit element operates N_c times during one call cycle with transfer velocity v, then the duration of its continuous operation with transferring contacts will equal

$$T_c = \frac{N_c}{3600v} \text{ hr}, \quad (3)$$

We assume that n_h call cycles are made during one hour. Then, the time of continuous element operation during one hour is defined by the expression

$$T_h = \frac{n_h N_c}{3600\, v} \text{ hr}, \quad (4)$$

Multiplying (4) by the number of hours in a year, we obtain the time of proper element operation during one year:

$$T_y = \frac{n_h N_c}{3600\, v} 8760 \text{ hr}, \quad (5)$$

From the design conditions, the useful life of the system must be not less than H years. During this period, the element under consideration must be found in the mode of continuous operation during

$$T' = HT_y = \frac{Hn_h N_c}{3600\, v} 8760 \text{ hr}, \quad (6)$$

where H is a dimensionless quantity expressing a quantity of years.

With the condition that

$$\frac{T_{adm}}{T'} \geqslant 1, \quad (7)$$

the given element operates properly until the end of the proposed time of proper functioning of the entire system. It is therefore not necessary to replace it. Otherwise, replacement would be necessary.

By substituting (1) and (6) in (7), we get

$$\frac{N_{adm}}{8760\, Hn_h N_c} \geqslant 1. \quad (7')$$

Whence,

$$n_h N_c \leqslant \frac{N_{adm}}{8760\, H}. \quad (8)$$

The left member of (8) is the number of transfers of the element in one hour. We call it the hourly intensity of the element's operation, and we use the notation

$$Q = n_h N_c.$$

Then, (8) can be given in the form

$$Q \leqslant \frac{N_{adm}}{8760\, H}, \quad (8')$$

Thus, we have established the possibility of proper element operation with a given N_{adm} as a function of the intensity of its operation. The appended figure gives the curve for $Q = \varphi(H)$, constructed from the condition of equality of the left and right members of (8'). It is therefore a boundary line: the region below it characterizes proper operation, and the upper region characterizes improper functioning of the element during the given period H. It should be mentioned that the character of variation of the curve $Q = \varphi(H)$ coincides with the character described by Feugel for the experimental reliability characteristic as a function of time [6]. This confirms the correctness of the initial premises of our analysis.

In a similar way, we can obtain the expression for estimating the useful life of reed frequency elements (Cf. the Appendix)

$$Q \leqslant \frac{N_{adm}}{8760\, H}\left(\frac{\tau}{t}\right). \quad (9)$$

It follows from this that, to increase the useful life of a reed element, tuned to frequency f, it is necessary to shorten the signal's transmission time t. This is possible only if the speed of response of the element itself is increased.

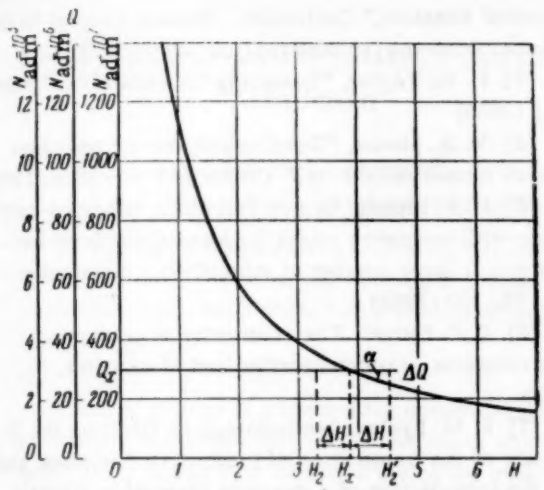
Estimating the usage state of contact apparatus by the method presented here is most effective in the case of mass-produced devices, since the value of N_{adm}, established just once on an experimental basis for individual elements and blocks, can be used as a basic parameter for all the apparatus produced in the same production run.

Influence of Useful Life of Apparatus on Its "Use" Expenditure

The economic efficiency of employing a system is usually estimated by the time taken to recoup its costs [7]

$$T = \frac{\Sigma K}{E_y}, \quad (10)$$

‡Here we have in mind parallel transmissions of frequency-indication pulses.



Boundary of the proper functioning characteristic of a relay contact element.

where Σ K is the total expenditure and E_y is the annual saving.

The total expenditure, together with the one-shot initial capital outlay and the wage fund for the personnel, includes deductions for amortization and maintenance, i.e., operating costs ("use" expenditures), a variable quantity which depends on maintenance and replacement periods of the system's individual elements and blocks.

Operating costs per element, in monetary units, we denote by K_A/A, where A is the total number of contact elements in the system. As a percentage of the sum of costs,** this quantity amounts to

$$K_A^* = \frac{K_A \Sigma K}{A \cdot 100}. \tag{11}$$

By substituting the value of ΣK from (10), we obtain

$$K_A^* = \frac{K_A}{100\,A} T E_y, \tag{11'}$$

whence,

$$T = \frac{K_A^* A \cdot 100}{K_A E_y}. \tag{a}$$

The condition for proper apparatus functioning during the time taken to recoup the system's cost is

$$T \leqslant H_x, \tag{12}$$

where H_x, for a given mode of element operation, is determined from the proper curve of $Q = \varphi(H)$. We assume that

$$T = H_x \pm \Delta H.$$

Obviously, element replacement is necessary if it is the plus sign which stands before ΔH. We now analyze how the quantity K_A^* varies with this.

From the figure, we have

$$\frac{\Delta Q}{\Delta H} = \tan\alpha \tag{13}$$

or

$$\frac{dQ}{dH} = \tan\alpha.$$

Bearing in mind the condition for the construction of the curves $Q = \varphi(H)$, we find from (8') that

$$\frac{dQ}{dH} = -\frac{N_{adm}}{H^2 \cdot 8760}. \tag{14}$$

By substituting the value of tan α in (13), we get

$$\Delta H = -\frac{\Delta Q H^2 \cdot 8760}{N_{adm}}. \tag{15}$$

If we adopt the convention that the values of K_A^* determined from (11') correspond to $H_x \geq T$ then, for $H_x < T$, the magnitude of the operating costs is expressed by the formula

$$K_{A1}^* = \frac{K_A}{100\,A} E_y H \left(1 + \frac{\Delta Q H}{N_{adm}} 8760\right). \tag{16}$$

The relative increment of operating costs per element, as a function of the mode and intensity of its operation, is defined by the expression

$$\Theta = \frac{K_{A1}^*}{K_A^*} = 1 + \frac{\Delta Q H}{N_{adm}} 8760. \tag{17}$$

We can, by a similar method, obtain the expression for the case when reed elements are used

$$\Theta = 1 + \frac{\Delta Q H}{N_{adm}} \left(\frac{t}{\tau}\right) 8760. \tag{18}$$

SUMMARY

1. The curves $Q = \varphi(H)$ for a system's individual elements and blocks allow timely replacement of elements on the verge of breakdown, and thereby reduce to a minimum the system's down time.

2. The expressions obtained permit the approximate estimation of and, if necessary, the correction of, initially assumed values for, the operating costs as applied to apparatus operating in identical conditions.

The development of a method for estimating the system as a whole is the subject of a special investigation.

The author takes this opportunity to express his profound gratitude to B. S. Sotskov for posing the problem and for giving his attention to its solution.

Appendix
Derivation of Expression (9)

The numbers of transfers of a frequency element during one call cycle is

$$N_c = ft.$$

If the given object is called n_h times an hour, then the total number of transfers of the element's contacts which participate in the selection of the given object, is

$$N_h = n_h N_c = n_h f t. \tag{19}$$

It is obvious that the quantity $n_h t = T_h$ is the duration of continuous transfer of the element's contacts during one hour. From (19),

** As in Russian. The correct percentage should be $\frac{K_A \cdot 100}{A \Sigma K}$ – Publisher.

$$T_h = \frac{N_c n_h}{f \cdot 3600} \text{ hr.} \qquad (20)$$

The time of continuous transfer of the element's contacts during one year is

$$T_y = \frac{N_c n_h}{f \cdot 3600} \ 8760 \text{ hr} \qquad (21)$$

Furthermore,

$$T' = HT_y = \frac{HN_c n_h}{f \cdot 3600} \ 8760 \text{ hr} \qquad (22)$$

By denoting $Q = N_c n_h = f t n_h$, and dividing (22) by (2), we obtain (9)

$$Q < \frac{N_{adm}}{H \cdot 8760} \left(\frac{\tau}{t}\right).$$

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A NUMERICAL-TO-ELECTRICAL CONVERTER

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The block diagram is described of a transducer which converts numerical quantities to proportional values of voltage or current, the conversion being based on the principle of an intermediate transformation to time intervals. The converter's operation is analyzed, and some general considerations relating to the design of such devices are given.

For the purpose of transforming numerical quantities to voltages or currents proportional to them, one can, in many cases, make very effective use of the principle of intermediate transformation of the numbers to time intervals. Below we describe a device based on this principle. Its operation is characterized by this, that in each transformation cycle one counter is used to develop two successive signals whose durations are such that the time for the entire cycle remains invariant while the duration of one of the signals is proportional to the value of the input quantity. This signal is used for modulating a stabilized dc voltage or current. After being smoothed, the duration-modulated voltage (current) is converted to the electrical equivalent of the numerical quantity.

To elucidate the principle of operation, we consider the block diagram shown in Fig. 1. It consists of the output register of a computer (triggers T_1-T_n), gates for uncomplemented code (g_1-g_n), gates for complemented code (g'_1-g'_n), a counter (triggers with counter inputs T_1-T_n), controlling trigger T, key K and smoothing device SD. The rectangles with the sketches of the bipolar pulses in them in Fig. 1 denote differentiating networks. One of the pulses in each rectangle has a stroke through it. This means that the pulses of this polarity will not henceforth be used.

The converter's operation is comprised of two cycles. In the first cycle, via gates g_1-g_n, the code for the number N to be converted is written in parallel in the counter. Pulses continuously applied to the counter's input with frequency f are added to the number written there, so that the counter is returned to its zero state at some moment of time. At this moment, an overflow pulse is emitted by its last stage (trigger T_n). The emission of the overflow pulse terminates the first cycle of operation.

At the beginning of the second cycle, the overflow pulse reverses the state of trigger (flip-flop) T, opening key K. Simultaneously with the key's opening, a signal from the differentiating network is applied to gates g'_1-g'_n, by means of which the number N is again written in the counter, but this time in complemented code. Input pulses again are added into the counter until it is reset to zero. At the moment when the counter returns to its zero state, a second overflow pulse is emitted, transferring T to its original state. This leads to the closing of the key and to the writing of the number N in uncomplemented form in the counter. After this, the aforementioned two-cycle process is repeated.

It is easily seen that the duration $\bar{\tau}$ of the first cycle is linearly related to the number $\bar{N}$, the complement of the number N, namely

$$\bar{\tau} = \frac{\bar{N} + 1}{f}. \qquad (1)$$

The equality just written is the mathematical expression of a well-known property of an n-place counter, staticizing input pulses in the number system, with radix b, which has been adopted. This property is formulated as follows. If some number is staticized in a counter, then the counter will be returned to its zero state after one adds to it a series of input pulses, the number of which would bring the counter total to b^n (i.e., the number of pulses is the 10's complement in the decimal system, the 2's complement in the binary system). This added number equals the complement of the given number (i.e., its 9's complement in the decimal system, its 1's complement in the binary system) plus one. Therefore, $\bar{\tau}$ can be defined as the product of $(\bar{N}+1)$ by the size of the interval between two neighboring input pulses.

As an example, we consider two special cases, when the numerical quantity to be converted, expressed in the binary system, has its maximum and minimum values. If $N = 0$ ($\bar{N} = 2^n - 1$), then the number written in the counter in the first cycle will equal zero. Consequently, the overflow pulse appears at the counter's output after 2^n input pulses, i.e., in the given case $\bar{\tau} = 2^n/f$. If the number to be transformed has its maximum value, that is, $N = 2^n - 1$ ($\bar{N} = 0$), then all the counter triggers will be in their "1" state at the beginning of the first cycle, and the application of the first input pulse will lead to the emission of the overflow pulse, i.e., for this case, $\bar{\tau} = 1/f$. We may show analogously that for any intermediate value, the magnitude of $\bar{\tau}$ is determined in accordance with (1).

The number written in the counter at the beginning of the second cycle equals $\bar{N}$. Therefore, the duration τ of the second cycle can be obtained from the expression

$$\tau = \frac{N + 1}{f}. \qquad (2)$$

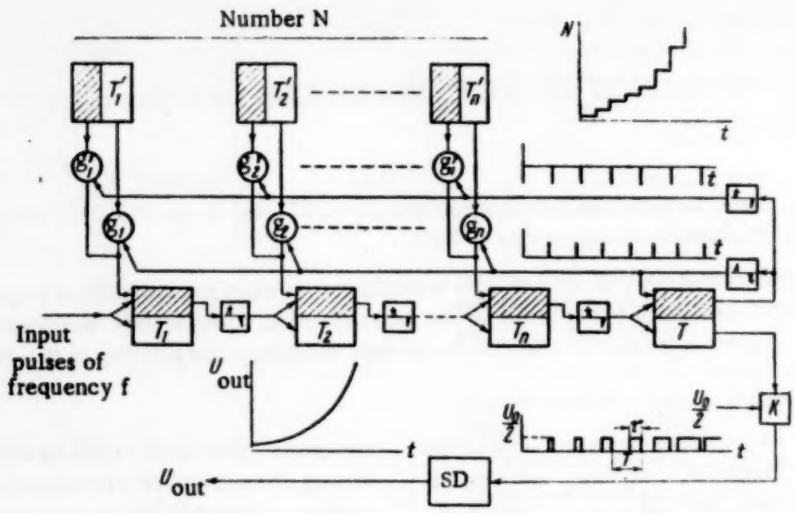

Fig. 1

On the basis of (1) and (2), the time T of a complete cycle, equal to the sum $\bar{\tau} + \tau$, is defined as

$$T = \frac{\overline{N} + N + 2}{f}. \qquad (3)$$

On the other hand, it is well known that the addition of an n-place binary number N to its complement $\overline{N}$ results in a constant which is independent of the absolute magnitude of N, namely:

$$N + \overline{N} = 2^n - 1 \qquad (4)$$

Consequently,

$$T = \frac{2^n + 1}{f}. \qquad (5)$$

On the basis of (5) and (2), we can draw the very important conclusion: in a converter executed in accordance with the block diagram of Fig. 1, the duration of a full cycle (composed of the two part cycles) does not depend on the value of the numerical quantity N to be converted. With this, each cycle consists of two part cycles; the duration of one of them, corresponding to the open state of key K, is a linear function of the number N.

The key K is connected in series in a current generator circuit, or in parallel to a dc voltage generator. In the first case, there will be at the smoothing device's input, duration-modulated current pulses and, in the second case, voltage pulses. Depending on which quantity is smoothed, the numerical quantity will be converted either to current or to voltage. For the sequel, this difference is immaterial so that, for the sake of definiteness, we shall assume that the number is converted to a voltage. Figure 2,a shows the curve of voltage variation at the smoothing device's input as a function of time u = f(t).

The output voltage U_{out} of the smoothing device is proportional to the mean value of the input voltage. If we denote the modulated dc voltage by $U_0/2$, we can write

$$U_{out} = \frac{U_0}{2}\frac{\tau}{T}. \qquad (6)$$

Or, taking (2) and (5) into account,

$$U_{out} = \frac{U_0}{2^{n+1}+2}(N+1). \qquad (7)$$

It is clear from (7) that the value of U_{out} is strictly proportional to the converted numerical quantity N. The constant term connected with the "1" in the parenthetical expression (N+1) can always be compensated, either by writing the number N-1, instead of N, in the computer's output register, or by biasing U_{out} by the quantity $U_0/(2^{n+1}+2)$.

Depending on the concrete requirements, the smoothing device can be executed in various ways. In particular, it may turn out to be advantageous to use the corresponding filters, special electronic devices or even simply pointer instruments indicating the average value of voltage (current). A consideration of all the possible practical variants

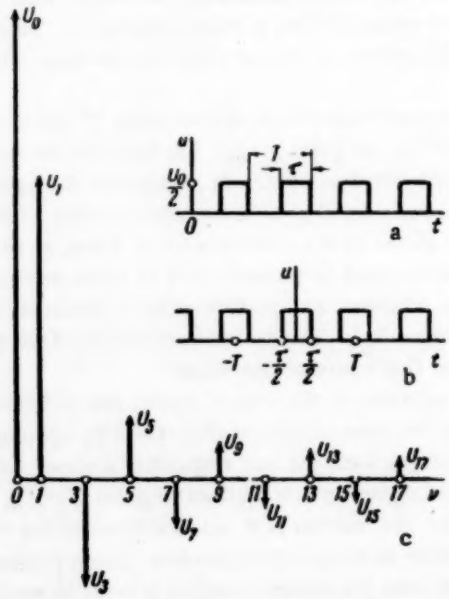

Fig. 2. The curve for u = f(t) and the frequency spectrum for θ = 0.5.

of the smoothing device is not part of our present task. It is relevant here to cite only several general considerations which should be kept in mind in designing it. For this, it is first necessary to look into the magnitude of the individual harmonic components of the curve $u = f(t)$.

In accordance with the converter's principle of operation, $u = f(t)$ for a given N is a periodic function. At any of its points it satisfies the Dirichlet condition, and may be expanded in a trigonometric series. We choose the origin of coordinates as is shown in Fig. 2,b. In this case, the function is an even one and, when expanded in a Fourier series, will have, in addition to the constant term defined by (6), only cosinusoidal terms. With this, the amplitude of the ν th harmonic U_ν is computed as the coefficient in the Fourier series

$$U_\nu = \frac{2}{T}\int_{-T/2}^{+T/2} f(t)\cos\frac{2\pi}{T}\nu t\,dt. \qquad (8)$$

If we bear in mind that the function f(t) has the constant value of $U_0/2$ in the interval $(-\tau/2+0) \le t \le (\tau/2-0)$ and equals zero in the intervals $-T/2 \le t \le (-\tau/2-0)$ and $(\tau/2+0) \le t \le T/2$, we can write Expression (8) in the form

$$U_\nu = \frac{U_0}{T}\int_{-\tau/2}^{+\tau/2} \cos\frac{2\pi}{T}\nu t\,dt. \qquad (9)$$

By carrying out the integration in (9), we get

$$U_\nu = \frac{U_0}{\pi\nu}\sin\pi\nu\frac{\tau}{T}. \qquad (10)$$

By denoting the duty factor, equal to the ratio τ/T, by θ we can write

$$U_\nu = \frac{U_0}{\pi\nu}\sin\pi\nu\theta. \qquad (11)$$

If we differentiate (11), we arrive at the expression

$$\frac{dU_\nu}{d\theta} = U_0\cos\pi\nu\theta. \qquad (12)$$

Equating the right member of (12) to zero:

$$U_0\cos\pi\nu\theta = 0, \qquad (13)$$

we can find the values of θ which correspond to maximum values of U_ν from the condition

$$\nu\theta = \frac{k}{2}, \qquad (14)$$

where k is any odd integer.

It must be mentioned that not all solutions which satisfy (14) should be taken into consideration. In fact, it is clear from (2) and (5) that

$$\theta = \frac{N+1}{2^n+1}. \qquad (15)$$

Since N is included in the interval

$$0 \leqslant N \leqslant 2^n - 1,$$

then

$$\frac{1}{2^n+1} \leqslant \theta \leqslant \frac{2^n}{2^n+1}. \qquad (16)$$

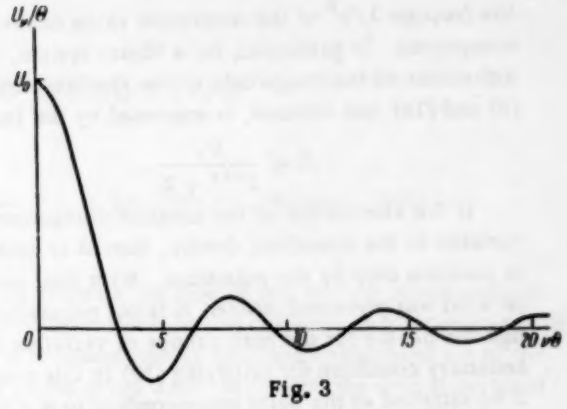

Fig. 3

Consequently, only those solutions of (14) have physical meaning for which the value of θ, for a given ν, satisfy (16). In particular, for $\nu = 1$, only the one value of θ equal to 0.5 is meaningful; for $\nu = 3$, the three values of θ, equal to 1/6, 1/2, and 5/6; for $\nu = 5$, the five values of θ equal to 0.1, 0.3, 0.5, 0.7, and 0.9, etc. It is characteristic of all this that there exists a unique value of θ, equal to 0.5, for which the peak voltage of all the harmonics, including the first, is a maximum. This can be confirmed by setting $\theta = 0.5$ in (14). After this substitution, we obtain the equation

$$\nu = k, \qquad (17)$$

which is satisfied for any odd ν. Therefore, the computations of the admissible pulsations at the output of the smoothing device should be carried out for the case $\theta = 0.5$.which is the most disadvantageous from the point of view of the size of the harmonic component. The frequency spectrum for this case, constructed on the basis of (6) and (11), is given in Fig. 2,c.

Figure 3 shows the envelope of the frequency spectrum for any physically meaningful values of ν and θ. Here, the quantity U_ν/θ has been laid out along the axis of ordinates, instead of U_ν, and the product $\nu\theta$, instead of ν, has been taken as the independent variable. As a result of these replacements, we can use one graph to show, in general form, the relationships between ordinal number of harmonic, magnitude of the individual components' amplitudes and the duty factors, which are proportional to the number N to be converted. The graph was constructed on the basis of (11), which can be written in the form

$$\frac{U_\nu}{\theta} = U_0\frac{\sin\pi\nu\theta}{\pi\nu\theta}. \qquad (18)$$

The harmonic components cause pulsations at the output of the smoothing device. In its turn, the size of the admissible pulsations is closely connected with the accuracy of the converter's operation. Indeed, if it is necessary to convert a numerical quantity into a voltage proportional to it, this then means that, over the entire range of variation of θ, the absolute error Δ at the smoothing device's output must not exceed the quantity which corresponds to one unit. In other words, Δ must be less than

the fraction $1/b^n$ of the maximum value of the constant component. In particular, for a binary system, the requirement on the magnitude of the absolute error, taking (6) and (16) into account, is expressed by the inequality

$$\Delta < \frac{U_0}{2^{n+1}+2}. \qquad (19)$$

If the attenuation of the constant component is invariable in the smoothing device, then Δ is determined in practice only by the pulsations. With this, and based on what was presented earlier, it is not necessary to design the device for the entire range of variation of θ. The necessary condition for satisfying (19) in this range is that it be satisfied at the point corresponding to $\theta = 0.5$.

In practical designing, it should be kept in mind that the speed of a concrete converter, for given f and n, is almost completely determined by the frequency properties of the smoothing device. These, in turn, depend on the device's form, circuit and parameters. With this, as the maximum number of significant places of the numerical-quantity converter decreases, the requirements on the permissible pulsations at the smoothing device's output, in correspondence with (19), relax sharply, which permits a significant increase in speed of action. The speed of the converter can also be increased by decreasing the duration T of a complete cycle which, as is clear from (5), is inversely proportional to the frequency f of the input pulses. But increasing f leads to the use of higher-frequency elements. Therefore, one should significantly increase the frequency of the input pulses only after one has exhausted the possibilities of increasing speed of action by improving the frequency properties of the smoothing device.

Among the advantages of the converter herein described are its simplicity, its lack of precision elements or assemblies, requiring fine adjustment, the ease of executing the device with transistor or ferro-transistor elements, the absence of special requirements on frequency stability of the input pulses, or on the duration of a complete cycle. This latter advantage is due to the fact that any variation of frequency automatically leads to an inversely proportional variation both of τ and of T and, in accordance with (6), thus has no effect on the magnitude of U_{out}.

AUTOMATIC TELEMETRY SYSTEM FOR OIL-WELL FLOW

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The basis for establishing the correct operating conditions for exploiting oil wells is the measurement, and taking account of, the quantity of oil and water extracted from the wells.

An optimal technological mode of operation of a well can only be established where there is sufficient information concerning the well's output and its variations under various operating modes. Variations in the output of fluid, observed by measurements, allow the necessary measures to be taken to guarantee normal well operation.

In the southern petroleum regions of the Soviet Union, flow measurements are made by grouped measuring stands. The use of such stands permits a sizeable shortening of the pipelines on the fields. At these stands, measurement of well flow is carried out manually. This increases the staffs of the fields, does not allow frequent measurements to be made (the measurers work on one shift) and does not permit systematic observation of well flow. Moreover, manual measurement of well flow retards the transition to complex automation and remote control of the fields.

In the present paper we describe a system of automatic telemetry of oil-well flow, developed at the Research and Planning Institute (NIPI) "Neftekhimavtomat."

The system developed for the automatic measurement of oil-well flow permits the measurement of oil and water flow individually from a well, with transmission of the readings to a control point and their recording on special cards. This system is one of the basic devices (assemblies) of the system of complex automation and remote (central) control of oil fields, developed jointly with the Institute for Automation and Remote Control (IAT) of the AN SSSR and the All-Union Petroleum Research Institute. In 1957, on the fourth field of the "Ordzhonikidzeneft' " oil-field management, workers of the fields and of the NIPI equipped 24 wells with this system, these wells being still successfully exploited today.

Figure 1 shows the control desk (a) and a measuring stand (b). Each measuring trap (measuring tank) serves eight wells (the number of wells can be increased).

Figure 2 shows the scheme of the system for the automatic telemetering of oil-well flow based on grouped measuring stands. Each such stand is equipped with distributor valves with pneumatic-membrane executive mechanisms (MEM) 8, a manometer 17, and position signaller (PS) 21.

To measure the flow of any well, it is necessary to switch it from the common collector to a measuring tank. This is implemented by applying compressed gas to the MEM of the given well. With this, the well is switched from common collector 10 to measuring tank 5, the MEM distributor valves are controlled by the pneumatic stepping distributor (PSD) which has 16 positions; eight of these correspond to a connection of a well to the measuring tank, and eight to an empty measuring tank. Details as to the position of the PSD are obtained by means of the pneumoelectric transformer 21. The PSD drive is a bellows mechanism.

When compressed gas is fed to the chamber of the PSD's bellows mechanism, the PSD is switched from the given position to the following one. The supply of compressed gas to the bellows mechanism is implemented by electropneumatic valve 16. When this latter receives an electric impulse, it sends gas to the bellows mechanism's chamber, thereby switching the PSD. In addition to the eight valves which switch the wells from the common collector to the measuring tank, there is common valve 7 for controlling the measuring tank's drain line. The MEM of this valve is subject to combined control: by the PSD and by electropneumatic valve (EPV) 20. The MEM of the valve on the drain line is maintained in the closed state during the period when a well is connected to the measuring tank by means of the PSD and, after the well is disconnected from the measuring tank, by means of EPV 20. To control the emptying of the measuring tank, zero-level indicator 6 is connected in its lower portion and, to signal overflow of the measuring tank, trouble indicator 14 is connected in. Compressed gas, obtained from the same measuring tank, is used to supply all the system's pneumatic elements. For the pressure stabilization and cleaning of the gas, the system contains pressure reducer 18 and filter 19. Measurement of the oil and water levels in the measuring tank is implemented by level transducer 1 with two floats, 3 and 4, inside of which constant magnets are placed. One of the floats rests on the "water-oil" dividing level, the other rests on the oil level. The floats freely move along the vertical brass tube (or a tube of some other nonmagnetic material) inside of which is to be found indicator 2, set in motion by the transducer's synchronous motor.

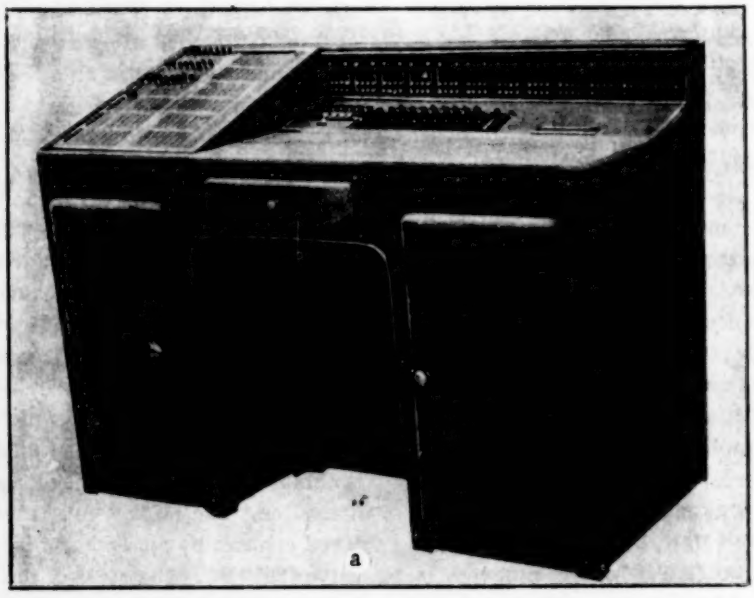
Control of the automated measuring tank's operation, and implementation of the interconnections between the stand's individual elements is carried out by

relay apparatus, located in the measuring tank's electrical block 13. At the central (control) point is to be found the control console, including program device 25, program-device commutator 26, relay apparatus 28, providing-automatic system operation, fixed frequency generator 29, flow recorder 30, and array 27 of signal lamps, keys and knobs.

The program device automatically gives the time for filling the measuring tanks by the output of each well in accordance with a previously established central control program within the limits of 10 minutes to 4 hrs, at 10 min intervals. The time for filling is established by the position of a contact pin on the program device commutator for each well individually. The programing device is built around stepping selectors. An individual stepping selector is assigned to each measuring tank.

The relay apparatus, consisting of several assemblies, connects the basic blocks and elements of the central control point, and provides automatic cyclic operation of the entire measuring system. The output recorder is a number-printing device for recording the serial number of the measuring tank, the well and the levels of oil and water. The electronic frequency generator is a two-channel one. There are 12 fixed frequencies in each channel.

In the system, a frequency method is used for selecting the objects (measuring tanks), and for transmission of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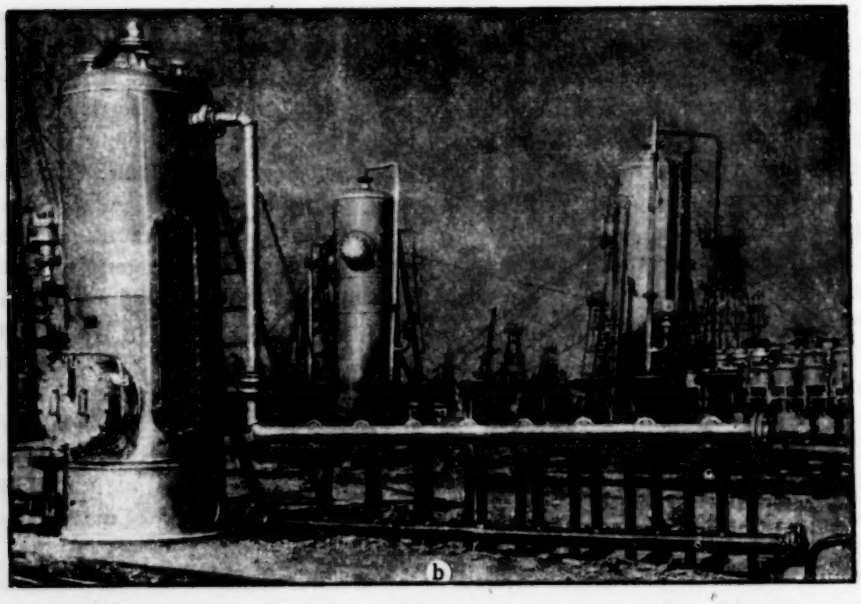
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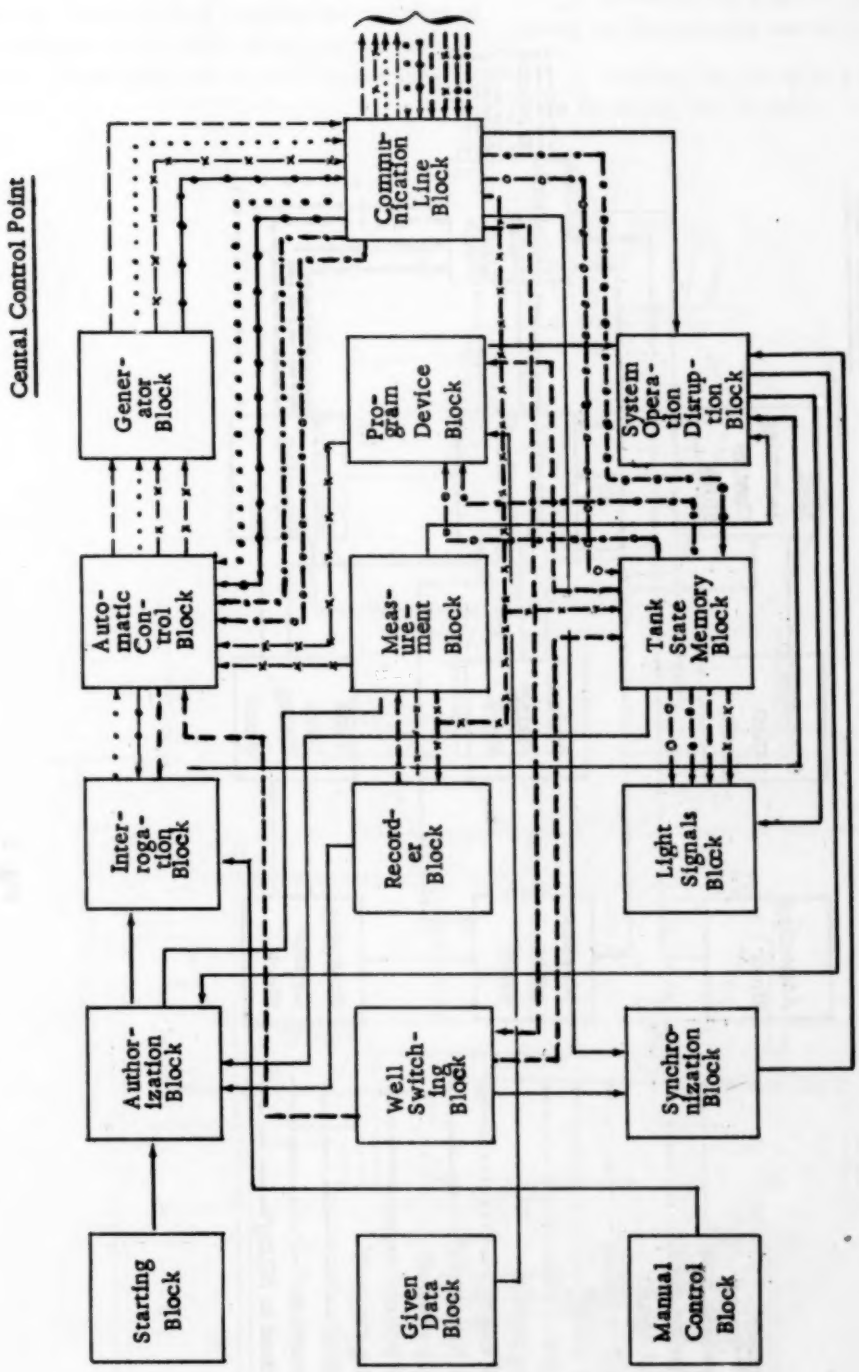
Fig. 1


Cental Control Point
Communication Line Block
Generator Block
Program Device Block
System Operation Disruption Block
Automatic Control Block
Measurement Block
Tank State Memory Block
Interrogation Block
Recorder Block
Light Signals Block
Authorization Block
Well Switching Block
Synchronization Block
Starting Block
Given Data Block
Manual Control Block

Legend:

1. Constant connection
2. "Interrogation signal"
3. Command signals
   a. "Switch to filling"
   b. "Stop filling"
   c. Measure
4. Answering signals
   a. "Tank empty"
   b. "Tank not empty"
   c. "Filling stopped" (liquid present)
   d. "Filling stopped" (no liquid)
   e. "Measurement"
   f. "Switched to filling"

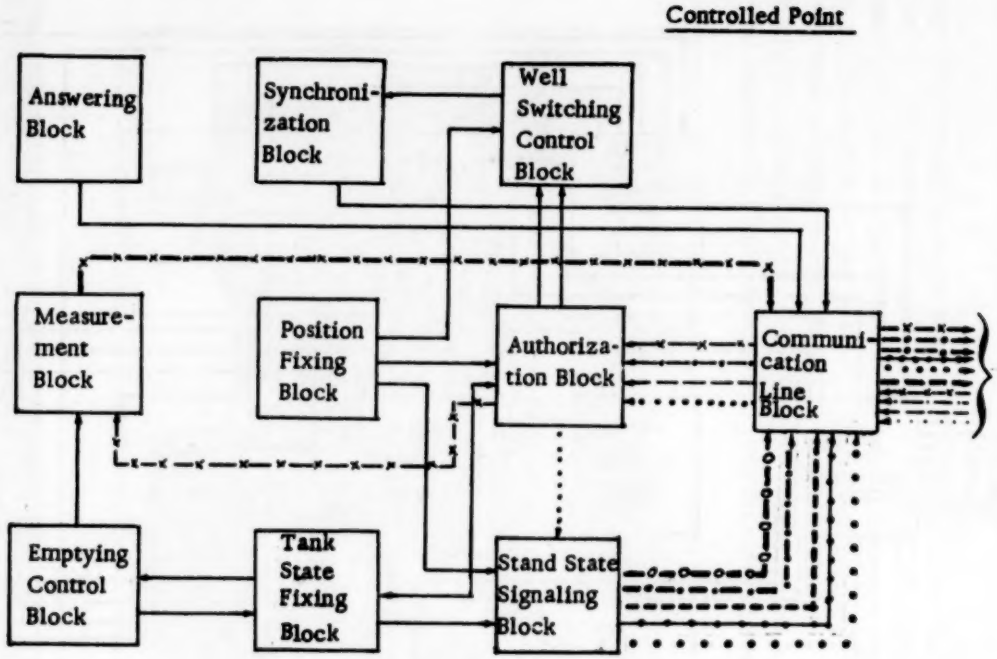


Fig. 3

the measurement signals, a pulse-time method of selection.

A given measuring stand is a continuously acting complex system, since each of the 12 measuring tanks comprising the system is, as it were, an independent unit, and has a closed operating cycle which is virtually independent of the state and operation of the other measuring tanks.

The system's functioning can be decomposed into several operations.

1. Interrogation as to the state of all the measuring tanks (periodically, at short intervals of from 2 to 5 minutes) with the aim of observing measuring tanks which are emptied subsequent to a prior measurement.

2. Switching an emptied measuring tank, observed during an interrogation period, to the filling mode.

3. Halting the filling of a measuring tank after the time for filling has elapsed.

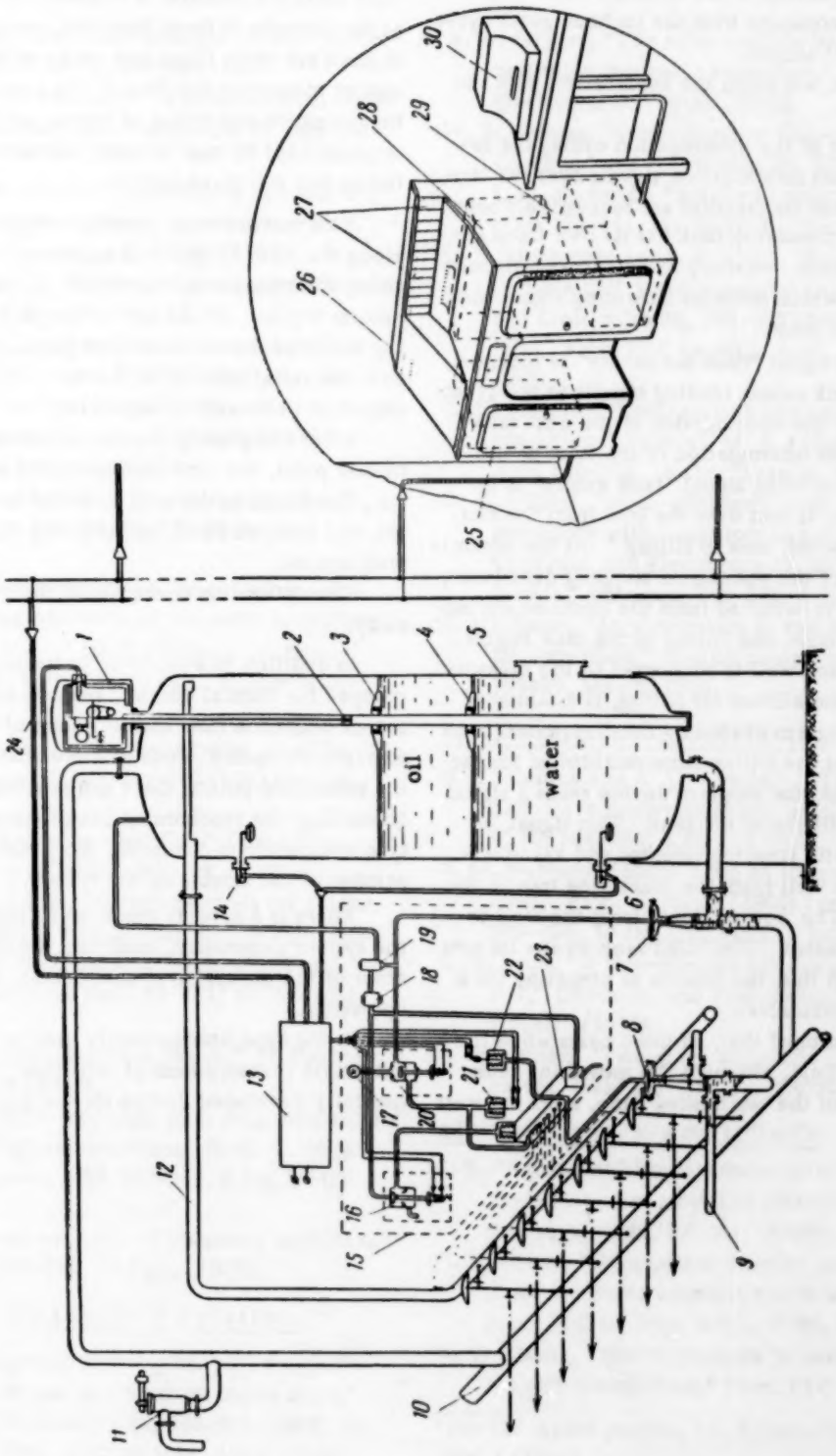


Fig. 2

4. Measuring and recording the results.

5. Switching to the emptying mode after completion of the measurement cycle.

Figure 3 shows the block schematic of the central control point and one of the 12 controlled points of the system of automatic telemetering of oil-well output. We shall consider the operation of the system as shown on the block diagram in accordance with the technological cycle of measuring oil-well output.

For convenience, we begin the explanation with the interrogation cycle.

At the beginning of the interrogation cycle, the interrogation block sends interrogation signals over the line, i.e., signals of 12 fixed frequencies are successively sent over the line. Each measuring tank has its own fixed selection frequency. Each frequency selects its corresponding measuring tank, which sends its answering signal back to the central control point.

If the answering signal "tank not empty" is received, the interrogation block ceases sending signals to the given tank, and switches to the interrogation of the next tank's state. This terminates interrogation of the state of the given tank. If the answering signal "tank empty" is received, then the order is sent over the line from the central control point, "switch tank to filling." At the controlled point, by means of the pneumatic stepping distributor, the next well in turn is switched from the common collector to the measuring tank, and filling of the tank begins.

The time that each well is connected to the measuring tank, i.e., the time alloted for filling, is established beforehand on the program device by the dispatcher (central controller). After the filling time established for the given well has elapsed, the program device sends a signal over the line to halt filling of the tank. This signal, by means of the pneumatic stepping selector and valve, effects switching of the well from the measuring trap to the common collector. The flow of liquid from the well to the measuring tank ceases. The filled tank awaits its turn to be measured. With this, the process of preparing for a flow measurement terminates.

It should be mentioned that, in those cases when the well does not supply fluid, although the measuring tank was in the fill state for the established time, after the well is switched from the measuring tank an answering signal, "filling stopped" (no fluid) is sent. This signal is fixed at the central control point and, when it again is time to interrogate the given tank, it is set up to be filled from the following well.

A measuring tank filled with liquid remains in that state until the recorder is released from operation. As soon as the recorder is freed from the previous measurement and if there are tanks filled and ready to be measured, the process of measuring the flow in the next tank begins. The measurement and fixing of results of the measurement is implemented by one recorder, common to all the tanks entering into the given system.

The measurement process is begun by the order, sent along the line, "begin measurement." At the controlled point, the transducer is switched in, and the measurement process begins. At the end of the process, the recorder, having received the measurement pulses, prints on a special card the serial number of the tank, the well and the well's output of water and oil separately.

After completing the measurement process at the controlled point, the tank automatically switches to emptying, i.e., the liquid in the tank is routed to the common collector, and the process of interrogating the state of the next tank begins.

Thereafter, the operation of the system repeats cyclically.

In addition to automatic operation, the system is equipped for manual control, manual calling of tanks and out-of-sequence flow measurement of any well, at the dispatcher's option. Both at the central control and at the controlled points, there are synchronization blocks, controlling the synchronization of system operation. If synchronization is disrupted, these blocks signal the dispatcher of the trouble in the system.

There is a special block for fixing the disruptions in the system's operation, notifying the dispatcher of the character of the disruption if such occurs, and preventing false operations.

In the apparatus presently used in the system for the automatic measurement of well flow, the recorder is a specially developed device for the given system.

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